When you see…

Find the zeros

You think…

1

To find the zeros...

Set function = 0

Factor *or* use quadratic equation if quadratic.

Graph to find zeros on calculator.

2

When you see…

Find equation of the line tangent to $f(x)$ at (a, b)

You think…

3

Equation of the tangent line

Point and Slope
(a, f(a)) f '(a)

Use $y - y_1 = f'(a)(x - x_1)$

4

When you see…

Find equation of the line normal to $f(x)$ at (a, b)

You think…

5

Equation of the normal line

Point and Slope
(a, f(a)) $m = -\frac{1}{f'(a)}$

Use $y - y_1 = m(x - x_1)$

6

When you see…

Show that $f(x)$ is even

You think…

7

Even function

$f(-x) = f(x)$

y-axis symmetry
(example: $f(x) = x^2$)

8

When you see…

Show that $f(x)$ is odd

You think…

9

Odd function

$f(-x) = -f(x)$

origin symmetry
(example: $f(x) = x^3$)

10

When you see…

Find the interval where $f(x)$ is increasing

You think…

11

f(x) increasing

Find $f'(x) = 0$ *or* $f'(x) =$ undef

Determine where $f'(x) > 0$

Answer: (a, b) or $a < x < b$

12

When you see…

Find the interval where the slope of $f(x)$ is increasing

You think…

13

Slope of f (x) is increasing

Find $f''(x)$

Set $f''(x) = 0$ and $f''(x) =$ undefined

Make sign chart of $f''(x)$

Determine where $f''(x)$ is positive

14

When you see…

Find the minimum value of a function

You think…

15

Local Minimum value of a function

Make a sign chart of $f'(x)$

Find x where $f'(x)$ changes from - to +

Plug those x values into f (x)

Choose the smallest

16

When you see…

Find critical numbers

You think…

17

Find critical numbers

Find **f ' (x)** = 0 and **f ' (x)** = undef.

18

When you see…

Find inflection points

You think…

19

Find inflection points

Find **f '' (x)** = 0 and **f '' (x)** = undef.

Make a sign chart of f **''** (x)

Find where **f '' (x)** changes sign
(+ to -) or (- to +)

20

When you see…

Show that $\lim_{x \to a} f(x)$ exists

You think…

21

Show $\lim_{x \to a} f(x)$ exists

Show that

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

22

When you see…

Show that $f(x)$ is continuous

You think…

23

f(x) is continuous

Show that

1) $\lim_{x \to a} f(x)$ exists (previous slide)

2) $f(a)$ exists

3) $\lim_{x \to a} f(x) = f(a)$

24

When you see…

Show that $f(x)$ is differentiable at x = a

You think…

25

f(x) is differentiable

Show that

1) f(a) is continuous (previous slide)

2) $\lim_{x \to a^-} f'(x) = \lim_{x \to a^+} f'(x)$

26

When you see…

Find vertical asymptotes of $f(x)$

You think…

27

Find vertical asymptotes of f(x)

Factor/cancel f(x)

Set denominator = 0

28

When you see…

Find horizontal asymptotes of $f(x)$

You think…

29

Find horizontal asymptotes of f(x)

Show

$$\lim_{x \to \infty} f(x)$$

and

$$\lim_{x \to -\infty} f(x)$$

30

When you see…

Find the average rate of change of $f(x)$ at $[a, b]$

You think…

31

Average rate of change of f(x)

Find

$$\frac{f(b) - f(a)}{b - a}$$

32

When you see…

Find the instantaneous rate of change of $f(x)$ at x = a

You think…

Instantaneous rate of change of f(x)

Find f ' (a)

When you see…

Find the average value of $f(x)$ on $[a, b]$

You think…

Average value of the function

Find $\frac{1}{b-a}\int_a^b f(x)dx$

When you see…

Find the absolute maximum of $f(x)$ on $[a, b]$

You think…

Find the absolute maximum of f(x)

a) Make a sign chart of f '(x)

b) Find all x values where relative maxima occur (where f '(x) changes from + to -) Be sure to check endpoints

c) Plug those x values into f (x)

d) Choose the largest y value

When you see…

Show that a piecewise function is differentiable at the point a where the function rule splits

You think…

Show a piecewise function is differentiable at x=a

First, be sure that the function is continuous at

$x = a$.

Take the derivative of each piece and show that

$\lim_{x\to a^-} f'(x) = \lim_{x\to a^+} f'(x)$

When you see…

Given $s(t)$ (position function), find $v(t)$

You think…

Given position s(t), find v(t)

Find $s'(t) = v(t)$

When you see…

Given $v(t)$, find how far a particle travels on [a, b]

You think…

Given v(t), find how far a particle travels on [a,b]

think . . . Total

Find $\int_a^b |v(t)|dt$

When you see…

Find the average velocity of a particle on [a, b]

You think…

Find the average rate of change on [a,b]

Find

$$\frac{\int_a^b v(t)\,dt}{b-a} = \frac{s(b)-s(a)}{b-a}$$

When you see…

Given $v(t)$, determine if a particle is speeding up at $t = k$

You think…

Given v(t), determine if the particle is speeding up at t=k

Find v(k) and a(k).

If v(k) and a(k) signs are the same, the particle is speeding up.

If v(k) and a(k) signs are different, the particle is slowing down.

When you see…

Given $v(t)$ and $s(0)$, find $s(t)$

You think…

Given v(t) and s(0), find s(t)

$$s(t) = s(0) + \int v(t)\,dt$$

When you see…

Show that the Mean Value Theorem holds on $[a, b]$

You think…

Show that the MVT holds on [a,b]

Show that f is continuous and differentiable on the interval.

Then find some c such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

When you see…

Show that Rolle's Theorem holds on $[a, b]$

You think…

Show that Rolle's Theorem holds on [a,b]

Show that f is continuous and differentiable on the interval

If $f(a) = f(b)$, then find some c in $[a, b]$ such that $f'(c) = 0$.

When you see…

Find the domain of $f(x)$

You think…

Find the domain of f(x)

Assume domain is $(-\infty, \infty)$.

Domain restrictions: non-zero denominators, Square root of non negative numbers, Log or ln of positive numbers

When you see…

Find the range of $f(x)$ on $[a, b]$

You think…

Find the range of f(x) on [a,b]

Use max/min techniques to find relative maximums and minimums.

Then examine f(a), f(b) and endpoints

When you see…

Find the range of $f(x)$ on $(-\infty, \infty)$

You think…

Find the range of f(x) on $(-\infty, \infty)$

Use max/min techniques to find relative max/mins.

Then examine $\lim_{x \to \pm\infty} f(x)$.

When you see…

Find $f'(x)$ by definition

You think…

Find f '(x) by definition

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \text{ or}$$

$$f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

When you see…

Find the derivative of the inverse of $f(x)$ at $x = a$

You think…

Derivative of the inverse of f(x) at x=a

Interchange x with y.

Find $\frac{dy}{dx}$ implicitly (in terms of y).

Plug your x value into the inverse relation and solve for y.

Finally, plug that y into your $\frac{dy}{dx}$.

When you see…

y is increasing proportionally to y

You think…

y is increasing proportionally to y

$$\frac{dy}{dt} = ky$$

translating to

$$y = Ce^{kt}$$

When you see…

Find the line $x = c$ that divides the area under $f(x)$ on $[a, b]$ into two equal areas

You think…

Find the x=c so the area under f(x) is divided equally

$$\int_a^c f(x)dx = \int_c^b f(x)dx$$

$$\int_a^c f(x)dx = \frac{1}{2}A$$

When you see…

$$\frac{d}{dx}\int_a^x f(t)dt =$$

You think…

Fundamental Theorem

2nd FTC: Answer is $f(x)$

When you see…

$$\frac{d}{dx}\int_a^m f(t)dt =$$

You think…

Fundamental Theorem, again

Given: $\frac{d}{dx}\int_a^m f(t)dt$

2nd FTC. . . Answer is: $f(m)\frac{dm}{dx}$

When you see…

The rate of change of population is …

You think…

Rate of change of a population

$$\frac{dP}{dt} = \ldots$$

When you see…

The line $y = mx + b$ is tangent to $f(x)$ at (a, b)

You think…

y = mx+b is tangent to f(x) at (a,b)

Two relationships are true.

The two functions share the same slope $(m = f'(x))$

and share the same point *(a,b)*

When you see…

Integrate

You think…

Methods for Integration

1. **Estimation:**
   - LRAM
   - RRAM (Riemann Sums)
   - MRAM
   - Trapezoid
2. **Geometry**
3. **Antiderivative**
   - Straight Forward
   - Substitution
   - Rewrite (Simplify)

When you see…

Find area using Left Riemann sums

You think…

Area using Left Riemann sums

$$A = \int_a^b R(t)dt$$

$$A \approx base\left[y_0 + y_1 + y_2 + \ldots + y_{n-1}\right]$$

When you see…

Find area using Right Riemann sums

You think…

81

Area using Right Riemann sums

$$A = \int_a^b R(t)\,dt$$

$$A \approx base\left[y_1 + y_2 + y_3 + \ldots + y_n\right]$$

82

When you see…

Find area using Midpoint rectangles

You think…

83

Area using midpoint rectangles

Typically done with a table of values.

Be sure to use only values that are given.

If you are given 6 sets of points, you can only do 3 midpoint rectangles.

84

When you see…

Find area using trapezoids

You think…

85

Area using trapezoids

$$A = \int_a^b R(t)\,dt$$

$$A \approx \frac{base}{2}\left[y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\right]$$

This formula only works when the base is the same.
If not, you have to do individual trapezoids.

86

When you see…

Solve the differential equation …

You think…

87

Solve the differential equation...

Separate the variables –

x on one side, y on the other.

Separate only by multiplication or division and the dx and dy must all be upstairs..

88

When you see…

Meaning of $\int_a^x f(t)\,dt$

You think…

89

Meaning of the integral of f(t) from a to x

The accumulation function –

accumulated area under the function $f(x)$

starting at some constant a and ending at x

90

When you see…

Given a base, cross sections perpendicular to the *x*-axis that are squares

You think…

91

Semi-circular cross sections perpendicular to the x-axis

The area between the curves typically is the base of your square.

$$V = \int_a^b \left(base\right)^2 dx$$

92

When you see…

Find where the tangent line to $f(x)$ is horizontal

You think…

93

Horizontal tangent line

Set $f'(x) = 0$

94

When you see…

Find where the tangent line to $f(x)$ is vertical

You think…

95

Vertical tangent line to f(x)

Find $f'(x)$.

Set the denominator equal to zero.

96

When you see…

Find the minimum acceleration given $v(t)$

You think…

97

Given v(t), find minimum acceleration

First find the acceleration $v'(t) = a(t)$

Minimize the acceleration by

- Setting $a'(t) = 0$ or $a'(t) =$ undef.
- Then determine where $a'(t)$ changes from – to +

98

When you see…

Approximate the value $f(0.1)$ of by using the tangent line to f at $x = a$

You think…

99

Approximate f(0.1) using tangent line to f(x) at x = 0

Find the equation of the tangent line to f using $y - y_1 = m(x - x_1)$

where $m = f'(a)$ and the point is $(a, f(a))$.

Then plug in 0.1 into x and solve for y.

Be sure to use an approximation (≈)sign.

100

When you see…

Given the value of $F(a)$ and the fact that the anti-derivative of f is F, find $F(b)$

You think…

101

Given $F(a)$ and the that the anti-derivative of f is F, find $F(b)$

Usually, this problem contains an antiderivative you cannot take. Utilize the fact that if $F(x)$ is the antiderivative of f,

then $\int_a^b f(x)dx = F(b) - F(a)$.

Solve for $F(b)$ using the calculator to find the definite integral

102

When you see…

Find the derivative of $f(g(x))$

You think…

103

Find the derivative of f(g(x))

$$f'(g(x)) \cdot g'(x)$$

Think . . . Chain Rule

104

When you see…

Given $\int_a^b f(x)dx$, find $\int_a^b [f(x) + k]dx$

You think…

105

Given area under a curve and vertical shift, find the new area under the curve

$$\int_a^b [f(x) + k]dx = \int_a^b f(x)dx + \int_a^b k dx$$

106

When you see…

Given a graph of $f'(x)$ find where $f(x)$ is increasing

You think…

107

Given a graph of f '(x) , find where f(x) is increasing

Make a sign chart of $f'(x)$

Determine where $f'(x)$ is positive

108

When you see…

Given $v(t)$ and $s(0)$, find the greatest distance from the origin of a particle on $[a, b]$

You think…

109

Given $v(t)$ and $s(0)$, find the greatest distance from the origin of a particle on $[a, b]$

Generate a sign chart of $v(t)$ to find turning points.

Integrate $v(t)$ using $s(0)$ to find the constant to find $s(t)$.

Find s(all turning points) which will give you the distance from your starting point.

Adjust for the origin.

110

When you see…

Given a water tank with g gallons initially being filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons/min on $[0, t_1]$, find

111

a) the amount of water in the tank at m minutes

You think…

112

Amount of water in the tank at t minutes

initial gallons $+ \int_0^{t_1} (F(t) - E(t))dt$

b) the rate the water amount is changing at m

You think…

Rate the amount of water is changing at t = m

$$\frac{d}{dt}\int_t^m (F(t) - E(t))dt = F(m) - E(m)$$

c) the time when the water is at a minimum

You think…

The time when the water is at a minimum

$F(m) - E(m) = 0,$

testing the endpoints as well.

When you see…

Given a chart of x and $f(x)$ on selected values between a and b, estimate $f'(x)$ where c is between a and b.

You think…

Straddle c, using a value k greater than c and a value h less than c.

so $f'(c) \approx \frac{f(k) - f(h)}{k - h}$

When you see…

Given $\frac{dy}{dx}$, draw a slope field

You think…

Draw a slope field of dy/dx

Use the given points

Plug them into $\frac{dy}{dx}$, drawing little lines with the indicated slopes at the points.

When you see…

Find the area between curves $f(x)$ and $g(x)$ on $[a,b]$

You think…

Area between f(x) and g(x) on [a,b]

$A = \int_a^b [f(x) - g(x)]dx$,

assuming $f(x) > g(x)$

When you see…

Find the volume if the area between the curves $f(x)$ and $g(x)$

with a representative rectangle ***perpendicular*** to the axis of rotation

You think…

Volume generated by rotating area between f(x) and g(x) with a representative rectangle ***perpendicular*** to the axis of rotation

$$V = \pi \int_a^b [R^2 - r^2]dx$$

When you see…

Find the volume if the area between the curves $f(x)$ and $g(x)$

with a representative rectangle ***parallel*** to the axis of rotation

You think…

Volume generated by rotating area between f(x) and g(x) with a representative rectangle ***parallel*** to the axis of rotation

$$V = 2\pi \int_a^b (x - axis)(rect)dx$$

Remember: Always . . . Big - small