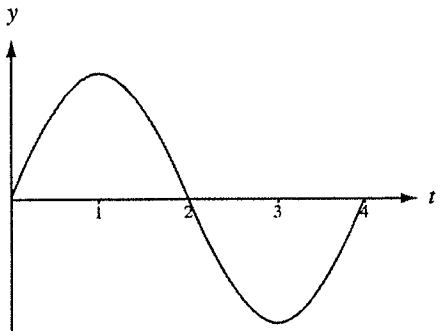
Name: _____

Sheet #610: **The Integral Function and the Fundamental Theorem**

The graph of the function $y=f(t)$ is shown below. The function is defined for $0 \le t \le 4$ and has the following properties:

- The graph of f has odd symmetry around the point $(2,0)$.
- On the interval $[0,2]$, the graph of f is symmetric with respect to the line $t=1$.
- $\int_0^1 f(t)\,dt = \frac{4}{3}$.



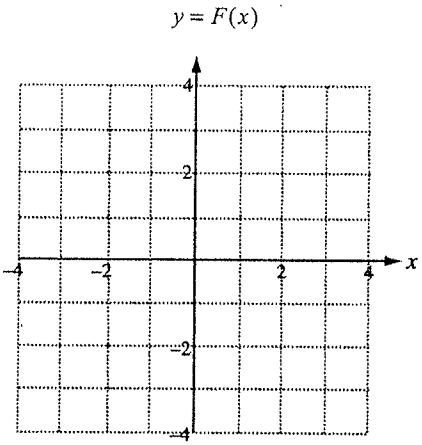
Graph of $y=f(t)$

1. Let $F(x) = \int_0^x f(t)\,dt$.

a. Complete the following table of values.

x	0	1	2	3	4
$F(x)$					

b. Sketch your best estimate of the graph of F on the grid below.

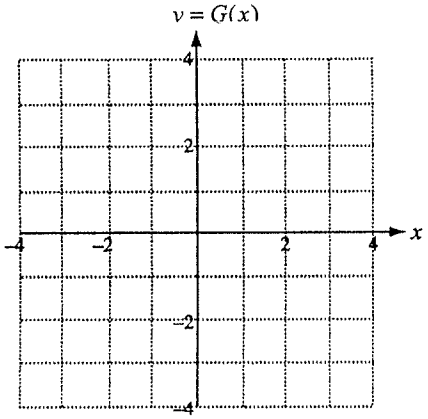


2. Let $G(x) = \int_2^x f(t)\,dt$.

a. Complete the following table of values.

x	0	1	2	3	4
$G(x)$					

b. Sketch the graph of G on the grid below.

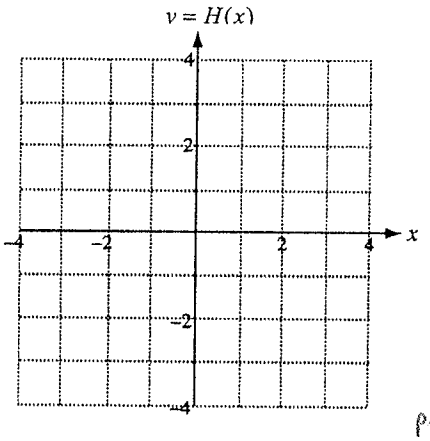


3. Let $H(x) = \int_4^x f(t)\,dt$.

a. Complete the following table of values.

x	0	1	2	3	4
$H(x)$					

b. Sketch the graph of H on the grid below.



4. Complete the following table.

F(x) G(x) H(x)

	$F(x)$	$G(x)$	$H(x)$
The maximum value of the function occurs at what x-value(s)?			
The minimum value of the function occurs at what x-value(s)?			
The function increases on what interval(s)?			
The function decreases on what interval(s)?			

5. Although the tables in questions 1, 2, and 3 asked only for the three functions to be evaluated at integer values of x, those functions were all continuous on the domain of $0 \le x \le 4$. Refer back to the answers you gave for function F in the table above, and explain why you believe each of these answers is correct when one considers F on its entire domain. Write your arguments in the table below. Your explanations should not rely on the graphs you sketched.

JUSTIFICATION FOR F(x)

	Justification of the answers above for $F(x)$
The maximum value of the function occurs at what x-value(s)?	
The minimum value of the function occurs at what x-value(s)?	
The function increases on what interval(s)?	
The function decreases on what interval(s)?	
... points of inflection	

6. What conjectures would you make about the family of functions of the form $W(x) = \int_k^x f(t)\,dt$ for $0 \le k \le 4$, where f is the graph given at the beginning of this worksheet?

THIS IS THE SAME FUNCTION...

The graph of the function $y=f(t)$ is shown below. The function is defined for $0 \le t \le 4$ and has the following properties:

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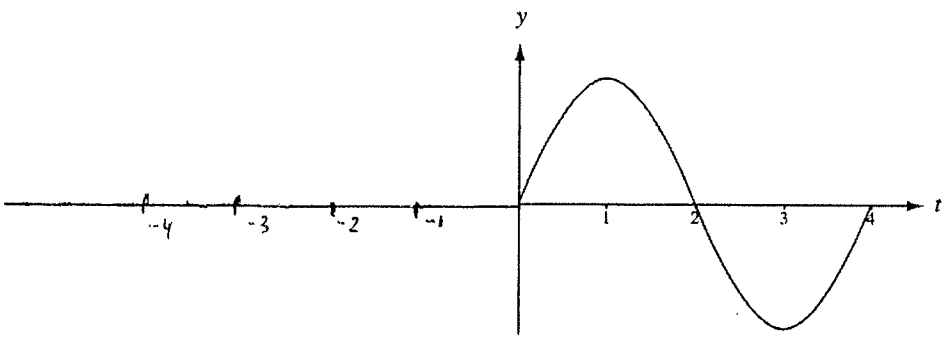
- On the interval $[0,2]$, the graph of f is symmetric with respect to the line $t=1$.
- $\int_0^1 f(t)\,dt = \frac{4}{3}$.

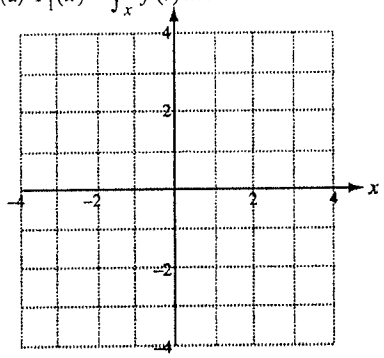


Graph of $y=f(t)$

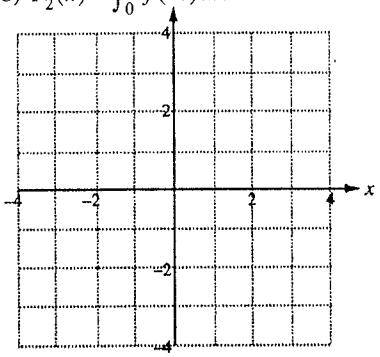
THESE ARE VARIATIONS... IT MAY BE HELPFUL TO MAKE SKETCHES ABOVE BEFORE INTEGRATING.

7. Extend your understanding by sketching each of the following functions.

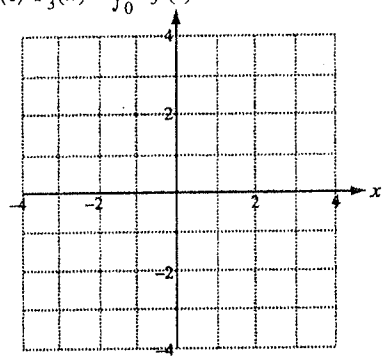
(a) $F_1(x) = \int_x^0 f(t)\,dt$.



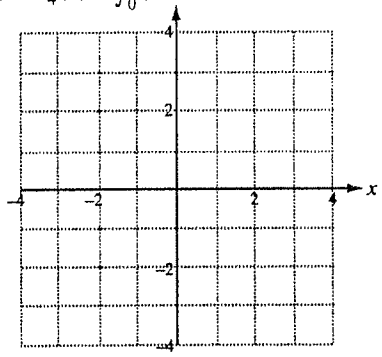
(b) $F_2(x) = \int_0^x f(-t)\,dt$.



(c) $F_3(x) = \int_0^{2x} f(t)\,dt$.



(d) $F_4(x) = \int_0^x f(|t|)\,dt$.



(e) $F_5(x) = \int_0^x |f(t)|\,dt$.