SHEET #354.
PRACTICE ON DIFFERENTIATION

NAME: KEY

PERIOD: ______

V.3

A. QUESTIONS 1 – 5, USE TABLE

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$	1b) $\frac{d}{dx}[f(g(x))]$	2b) $\frac{d}{dx}[g(f(x))]$
0	2	5	3	9	$f'(g(0))\cdot g'(0)=f'(3)\cdot 9=-2\cdot 9=\boxed{-18}$	$g'(2)\cdot 5=\boxed{70}$
1	3	4	0	11	$f'(g(1))\cdot g'(1)=f'(0)\cdot 11=5\cdot 11=\boxed{55}$	$g'(3)\cdot 4=\boxed{80}$
2	0	3	2	14	$f'(2)\cdot 14=3\cdot 14=\boxed{42}$	$g'(0)\cdot 3=\boxed{27}$
3	1	−2	1	20	$f'(1)\cdot 20=\boxed{80}$	$g'(1)\cdot -2=\boxed{-22}$

SORRY: THE DERIVATIVES ARE UNLIKELY.

1a, WRITE FORMULA FOR $\frac{d}{dx}[f(g(x))] = \boxed{f'(g(x))\cdot g'(x)}$

b, FILL OUT TABLE. SHOW WORK.

2a, WRITE FORMULA FOR $\frac{d}{dx}[g(f(x))] = \boxed{g'(f(x))\cdot f'(x)}$

b, FILL OUT TABLE. SHOW WORK.

DECOMPOSE: $f=\frac{1}{x^2}$ $f'=\frac{-2}{x^3}$

3. LET $R(x)=\frac{1}{[g(x)]^2}$. FIND $R'(0) = \frac{-2}{[g(0)]^3}\cdot g'(0) = \frac{-2}{(3)^3}\cdot 9=\boxed{\frac{-2}{3}}$

4. LET $P(x)=f(3x)$. FIND $P'(1) = f'(3\cdot 1)\cdot 3=(-2)(3)=\boxed{-6}$

NOTE: $\frac{d}{dx}(3x)=3$.

5. LET $Q(x) = f(x^4)$. FIND $Q'(1)$.

DECOMPOSE: $g(x)=x^4$ $g'(x)=4x^3$

$Q'(x) = f'(x^4)\cdot 4\cdot x^3$. $Q'(1)=f'(1^4)\cdot 4\cdot 1^3$

$= f'(1)\cdot 4 = 4\cdot 4=\boxed{16}$

B. MISCELLANEOUS PROBLEMS.

6. $\frac{d}{dx}[x^2\cos(x)] = 2x\cos x + x^2(-\sin x) = \boxed{x(2\cos x - x\sin x)}$

7. $\frac{d^2}{dx^2}(\sin(4x))$. $\frac{d}{dx}\sin(4x) = \cos(4x)\cdot 4$. $\frac{d}{dx}(\cos(4x)\cdot 4) = -\sin(4x)\cdot 4\cdot 4$.

$\rightarrow$ ANSWER $=\boxed{-16\sin(4x)}$

8. $\frac{d}{dx}\left(e^{\cos(x/4)}\right)$. TRIPLE CHAIN RULE. $\frac{d}{dx}\cos(x/4) = -\sin(x/4)\cdot\frac{d}{dx}\left(\frac{x}{4}\right)$.

DECOMPOSE: $f(x)=e^x$ $f'(x)=e^x$

$g(x)=\cos(x/4)$ $g'(x) = -\sin(x/4)\cdot\frac{1}{4}$

$\rightarrow$ ANSWER $= e^{g(x)}\cdot g'(x) = e^{\cos(x/4)}\cdot -\sin(x/4)\cdot\frac{1}{4}$

$= \boxed{-\frac{1}{4}\sin\left(\frac{x}{4}\right)\cdot e^{\cos(x/4)}}$

9. FIND THE TANGENT LINE TO $f(x)=\sqrt{x^2+1}$ AT $x=1$.

TANGENT LINE: $y - f(1) = f'(1)(x-1)$

$f(1)=\sqrt{1^2+1}=\sqrt{2}$

CHAIN RULE

DECOMPOSE $\begin{cases} f(x)=\sqrt{x} & f'(x)=\frac{1}{2\sqrt{x}} \\ g(x)=x^2+1 & g'(x)=2x \end{cases}$

$\frac{d}{dx}\sqrt{x^2+1} = \frac{1}{2\sqrt{x^2+1}}\cdot 2x$

$f'(1) = \frac{1}{2\sqrt{1^2+1}}\cdot 2\cdot 1 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\boxed{y-\sqrt{2} = \frac{\sqrt{2}}{2}(x-1)}$ $y=\frac{\sqrt{2}}{2}x+\frac{\sqrt{2}}{2}$

10. $\frac{d}{dx}\left(\sqrt[5]{x}\right) = \frac{d}{dx}\left(x^{1/5}\right) = \frac{1}{5}x^{-4/5} = \boxed{\frac{1}{5x^{4/5}}} = \frac{1}{5(\sqrt[5]{x})^4}$