Section 4.2 Exercises MEAN VALUE THEOREM. BOOK: FINNEY DEMANA (2003)

In Exercises 1–8, use analytic methods to find **(a)** the local extrema, **(b)** the intervals on which the function is increasing, and **(c)** the intervals on which the function is decreasing.

1. $f(x) = 5x - x^2$

2. $g(x) = x^2 - x - 12$

3. $h(x) = \dfrac{2}{x}$

4. $k(x) = \dfrac{1}{x^2}$

5. $f(x) = e^{2x}$

6. $f(x) = e^{-0.5x}$

7. $y = 4 - \sqrt{x+2}$

8. $y = x^4 - 10x^2 + 9$

In Exercises 9–14, find **(a)** the local extrema, **(b)** the intervals on which the function is increasing, and **(c)** the intervals on which the function is decreasing.

9. $f(x) = x\sqrt{4-x}$

10. $g(x) = x^{1/3}(x+8)$

11. $h(x) = \dfrac{-x}{x^2+4}$

12. $k(x) = \dfrac{x}{x^2-4}$

13. $f(x) = x^3 - 2x - 2\cos x$

14. $g(x) = 2x + \cos x$

In Exercises 15–18, **(a)** show that the function f satisfies the hypotheses of the Mean Value Theorem on the given interval $[a, b]$. **(b)** Find each value of c in (a, b) that satisfies the equation

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

15. $f(x) = x^2 + 2x - 1, \quad [0, 1]$

16. $f(x) = x^{2/3}, \quad [0, 1]$

17. $f(x) = \sin^{-1} x, \quad [-1, 1]$

18. $f(x) = \ln(x - 1), \quad [2, 4]$

In Exercises 19 and 20, the interval $a \le x \le b$ is given. Let $A = (a, f(a))$ and $B = (b, f(b))$. Write an equation for

(a) the secant line AB.

(b) a tangent line to f in the interval (a, b) that is parallel to AB.

19. $f(x) = x + \dfrac{1}{x}, \quad 0.5 \le x \le 2$

20. $f(x) = \sqrt{x-1}, \quad 1 \le x \le 3$

In Exercises 21–24, **(a)** show that the function f does not satisfy the hypotheses of the Mean Value Theorem on the given interval $[a, b]$. **(b)** Graph f together with the line through the points $A(a, f(a))$ and $B(b, f(b))$. **(c)** Find any values of c in (a, b) that satisfy the equation

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

21. $f(x) = x^{1/3}, \quad [-1, 1]$

22. $f(x) = |x - 1|, \quad [0, 3]$

23. $f(x) = 1 - |x|, \quad [-1, 1]$

24. $f(x) = \begin{cases} \cos x, & -\pi \le x < 0, \\ 1 + \sin x, & 0 \le x \le \pi, \end{cases}$ on $[-\pi, \pi]$

In Exercises 25–30, find all possible functions f with the given derivative.

25. $f'(x) = x$

26. $f'(x) = 2$

27. $f'(x) = 3x^2 - 2x + 1$

28. $f'(x) = \sin x$

29. $f'(x) = e^x$

30. $f'(x) = \dfrac{1}{x-1}, \quad x > 1$

In Exercises 31–34, find the function with the given derivative whose graph passes through the point P.

31. $f'(x) = -\dfrac{1}{x^2}, \quad x > 0, \quad P(2, 1)$

32. $f'(x) = \dfrac{1}{4x^{3/4}}, \quad P(1, -2)$

33. $f'(x) = \dfrac{1}{x+2}, \quad x > -2, \quad P(-1, 3)$

34. $f'(x) = 2x + 1 - \cos x, \quad P(0, 3)$

In Exercises 35–38, *work in groups of two or three* and sketch a graph of a differentiable function $y = f(x)$ that has the given properties.

35. **(a)** local minimum at (1, 1), local maximum at (3, 3)

(b) local minima at (1, 1) and (3, 3)

(c) local maxima at (1, 1) and (3, 3)

36. $f(2) = 3, \quad f'(2) = 0,$ and

(a) $f'(x) > 0$ for $x < 2, \quad f'(x) < 0$ for $x > 2$.

(b) $f'(x) < 0$ for $x < 2, \quad f'(x) > 0$ for $x > 2$.

(c) $f'(x) < 0$ for $x \ne 2$.

(d) $f'(x) > 0$ for $x \ne 2$.

37. $f'(-1) = f'(1) = 0, \quad f'(x) > 0$ on $(-1, 1)$, $f'(x) < 0$ for $x < -1, \quad f'(x) > 0$ for $x > 1$.

38. A local minimum value that is greater than one of its local maximum values

39. *Speeding* A trucker handed in a ticket at a toll booth showing that in 2 h she had covered 159 mi on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why?

40. *Temperature Change* It took 20 sec for the temperature to rise from 0°F to 212°F when a thermometer was taken from a freezer and placed in boiling water. Explain why at some moment in that interval the mercury was rising at exactly 10.1°F/sec.

41. *Triremes* Classical accounts tell us that a 170-oar trireme (ancient Greek or Roman warship) once covered 184 sea miles in 24 h. Explain why at some point during this feat the trireme's speed exceeded 7.5 knots (sea miles per hour).

42. *Running a Marathon* A marathoner ran the 26.2-mi New York City Marathon in 2.2 h. Show that at least twice, the marathoner was running at exactly 11 mph.

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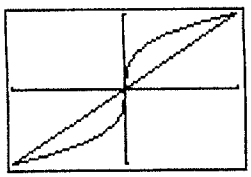
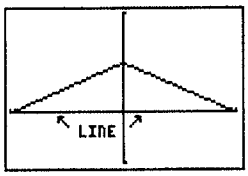
4. For all x in its domain, or, $[-2, 2]$
5. On $(-2, 2)$ **6.** $x \neq \pm 1$
7. For all x in its domain, or, for all $x \neq \pm 1$
8. For all x in its domain, or, for all $x \neq \pm 1$
9. $C = 3$ **10.** $C = -4$

Exercises ANSWERS 4-2 (MVT).

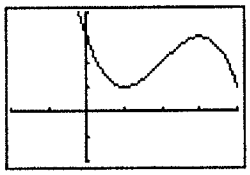
1. **(a)** Local maximum at $\left(\frac{5}{2}, \frac{25}{4}\right)$
(b) On $\left(-\infty, \frac{5}{2}\right]$ **(c)** On $\left[\frac{5}{2}, \infty\right)$
3. **(a)** None **(b)** None
(c) On $(-\infty, 0)$ and $(0, \infty)$
5. **(a)** None **(b)** On $(-\infty, \infty)$
(c) None
7. **(a)** Local maximum at $(-2, 4)$
(b) None **(c)** On $[-2, \infty)$
9. **(a)** Local maximum at $\approx (2.67, 3.08)$; local minimum at $(4, 0)$
(b) On $\left(-\infty, \frac{8}{3}\right]$ **(c)** On $\left[\frac{8}{3}, 4\right]$
11. **(a)** Local maximum at $\left(-2, \frac{1}{4}\right)$; local minimum at $\left(2, -\frac{1}{4}\right)$
(b) On $(-\infty, -2]$ and $[2, \infty)$
(c) On $[-2, 2]$
13. **(a)** Local maximum at $\approx (-1.126, -0.036)$; local minimum at $\approx (0.559, -2.639)$
(b) On $(-\infty, -1.126]$ and $[0.559, \infty)$
(c) On $[-1.126, 0.559]$
15. **(a)** f is continuous on $[0, 1]$ and differentiable on $(0, 1)$.
(b) $c = \frac{1}{2}$
17. **(a)** f is continuous on $[-1, 1]$ and differentiable on $[-1, 1]$.
(b) $c \approx \pm 0.771$
19. **(a)** $y = \frac{5}{2}$ **(b)** $y = 2$
21. **(a)** Not differentiable at $x = 0$
(b)



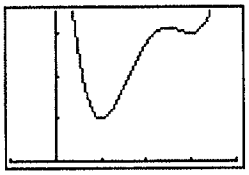
$[-1, 1]$ by $[-1, 1]$
(c) $c = \pm 3^{-3/2} \approx \pm 0.192$
23. **(a)** Not differentiable at $x = 0$
(b)



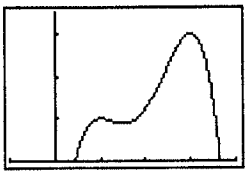
$[-1, 1]$ by $[-1, 2]$
(c) There are none
25. $\frac{x^2}{2} + C$ **27.** $x^3 - x^2 + x + C$
29. $e^x + C$ **31.** $\frac{1}{x} + \frac{1}{2}, x > 0$
33. $\ln(x + 2) + 3$
35. Possible answers:
(a)



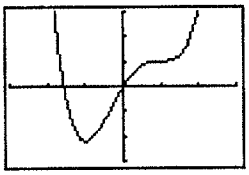
$[-2, 4]$ by $[-2, 4]$
(b)



$[-1, 4]$ by $[0, 3.5]$
(c)



$[-1, 4]$ by $[0, 3.5]$
37. One possible answer:



$[-3, 3]$ by $[-15, 15]$
39. Because the trucker's average speed was 79.5 mph, and by the Mean Value Theorem, the trucker must have been going that speed at least once during the trip.
41. Because its average speed was approximately 7.667 knots, and by the Mean Value Theorem, it must have been going that speed at least once during the trip.
43. **(a)** 48 m/sec **(b)** 720 meters
(c) After about 27.604 seconds, and it will be going about 48.166 m/sec
45. Because the function is not continuous on $[0, 1]$.

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