Test Review for Chapter 3.1 – 3.5 (in Hughes Hallett) *from Chapters 2.3, 2.4 and 5.4 in Larson*

Questions 2.3

→ O = DO ALL

O = OPTIONAL

REVIEW

2.3

(1.)

Lab Series LAB 3

EXERCISES FOR SECTION 2.3

In Exercises 1–6, use the Product Rule to differentiate the function.

→ **1.** $g(x) = (x^2 + 1)(x^2 - 2x)$

2. $f(x) = (6x + 5)(x^3 - 2)$

3. $h(t) = \sqrt[3]{t}(t^2 + 4)$

4. $g(s) = \sqrt{s}(4 - s^2)$

5. $f(x) = x^3 \cos x$

6. $g(x) = \sqrt{x} \sin x$

In Exercises 7–12, use the Quotient Rule to differentiate the function.

→ **7.** $f(x) = \dfrac{x}{x^2 + 1}$

8. $g(t) = \dfrac{t^2 + 2}{2t - 7}$

9. $h(x) = \dfrac{\sqrt[3]{x}}{x^3 + 1}$

10. $h(s) = \dfrac{s}{\sqrt{s} - 1}$

→ **11.** $g(x) = \dfrac{\sin x}{x^2}$

12. $f(t) = \dfrac{\cos t}{t^3}$

In Exercises 13–18, find $f'(x)$ and $f'(c)$.

	Function	*Value of c*
13.	$f(x) = (x^3 - 3x)(2x^2 + 3x + 5)$	$c = 0$
14.	$f(x) = (x^2 - 2x + 1)(x^3 - 1)$	$c = 1$
15.	$f(x) = \dfrac{x^2 - 4}{x - 3}$	$c = 1$
16.	$f(x) = \dfrac{x + 1}{x - 1}$	$c = 2$
→ **17.**	$f(x) = x \cos x$	$c = \dfrac{\pi}{4}$
18.	$f(x) = \dfrac{\sin x}{x}$	$c = \dfrac{\pi}{6}$

In Exercises 19–24, complete the table without using the Quotient Rule (see Example 6).

WITHOUT

	Function	*Rewrite*	*Differentiate*	*Simplify*
19.	$y = \dfrac{x^2 + 2x}{3}$			
20.	$y = \dfrac{5x^2 - 3}{4}$			
21.	$y = \dfrac{7}{3x^3}$			
22.	$y = \dfrac{4}{5x^2}$			
23.	$y = \dfrac{4x^{3/2}}{x}$			
24.	$y = \dfrac{3x^2 - 5}{7}$			

In Exercises 25–38, find the derivative of the algebraic function.

25. $f(x) = \dfrac{3 - 2x - x^2}{x^2 - 1}$

26. $f(x) = \dfrac{x^3 + 3x + 2}{x^2 - 1}$

27. $f(x) = x\left(1 - \dfrac{4}{x + 3}\right)$

28. $f(x) = x^4\left(1 - \dfrac{2}{x + 1}\right)$

→ **29.** $f(x) = \dfrac{2x + 5}{\sqrt{x}}$

30. $f(x) = \sqrt[3]{x}(\sqrt{x} + 3)$

31. $h(s) = (s^3 - 2)^2$

32. $h(x) = (x^2 - 1)^2$

→ **33.** $f(x) = \dfrac{2 - \frac{1}{x}}{x - 3}$

34. $g(x) = x^2\left(\dfrac{2}{x} - \dfrac{1}{x + 1}\right)$

35. $f(x) = (3x^3 + 4x)(x - 5)(x + 1)$

36. $f(x) = (x^2 - x)(x^2 + 1)(x^2 + x + 1)$

37. $f(x) = \dfrac{x^2 + c^2}{x^2 - c^2}$, c is a constant

38. $f(x) = \dfrac{c^2 - x^2}{c^2 + x^2}$, c is a constant

In Exercises 39–54, find the derivative of the trigonometric function.

→ **39.** $f(t) = t^2 \sin t$

40. $f(\theta) = (\theta + 1) \cos \theta$

41. $f(t) = \dfrac{\cos t}{t}$

42. $f(x) = \dfrac{\sin x}{x}$

43. $f(x) = -x + \tan x$

44. $y = x + \cot x$

45. $g(t) = \sqrt[4]{t} + 8 \sec t$

46. $h(s) = \dfrac{1}{s} - 10 \csc s$

→ **47.** $y = \dfrac{3(1 - \sin x)}{2 \cos x}$

48. $y = \dfrac{\sec x}{x}$

49. $y = -\csc x - \sin x$

50. $y = x \sin x + \cos x$

51. $f(x) = x^2 \tan x$

52. $f(x) = \sin x \cos x$

53. $y = 2x \sin x + x^2 \cos x$

54. $h(\theta) = 5\theta \sec \theta + \theta \tan \theta$

In Exercises 55–58, use a computer algebra system to differentiate the function.

55. $g(x) = \left(\dfrac{x + 1}{x + 2}\right)(2x - 5)$

56. $f(x) = \left(\dfrac{x^2 - x - 3}{x^2 + 1}\right)(x^2 + x + 1)$

57. $g(\theta) = \dfrac{\theta}{1 - \sin \theta}$

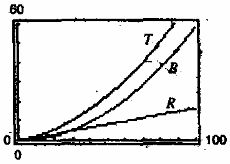
58. $f(\theta) = \dfrac{\sin \theta}{1 - \cos \theta}$

In Exercises 59–62, evaluate the derivative of the function at the indicated point. Use a graphing utility to verify your result.

	Function	*Point*
59.	$y = \dfrac{1 + \csc x}{1 - \csc x}$	$\left(\dfrac{\pi}{6}, -3\right)$
60.	$f(x) = \tan x \cot x$	$(1, 1)$
61.	$h(t) = \dfrac{\sec t}{t}$	$\left(\pi, -\dfrac{1}{\pi}\right)$
62.	$f(x) = \sin x(\sin x + \cos x)$	$\left(\dfrac{\pi}{4}, 1\right)$

2.3 (2)

99. (a) $R(v) = 0.167v - 0.02$
(b) $B(v) = 0.006v^2 - 0.024v + 0.460$
(c) $T(v) = 0.006v^2 + 0.143v + 0.440$
(d)



(e) $T'(v) = 0.012v + 0.143$
$T'(40) = 0.623$
$T'(80) = 1.103$
$T'(100) = 1.343$
(f) Stopping distance increases at an increasing rate.

101. 8 square meters per meter change in s

103. $-\$1.91, -\$1.93$

105. (a) The rate of change of gallons of gasoline sold when the price is \$1.479
(b) In general, the rate of change when $p = 1.479$ should be negative.

107. $y = 2x^2 - 3x + 1$ **109.** $y = -9x, y = -\frac{9}{4}x - \frac{27}{4}$

111. $a = \frac{1}{3}, b = -\frac{4}{3}$ **113.** Proof

Section 2.3 (page 124)

1. $2(2x^3 - 3x^2 + x - 1)$ **3.** $\dfrac{7t^2 + 4}{3t^{2/3}}$

5. $x^2(3\cos x - x\sin x)$ **7.** $\dfrac{1 - x^2}{(x^2 + 1)^2}$

9. $\dfrac{1 - 8x^3}{3x^{2/3}(x^3 + 1)^2}$ **11.** $\dfrac{\cancel{x\cos x} - 2\sin x}{x^3}$

13. $f'(x) = (x^3 - 3x)(4x + 3) + (2x^2 + 3x + 5)(3x^2 - 3)$
$= 10x^4 + 12x^3 - 3x^2 - 18x - 15$
$f'(0) = -15$

15. $f'(x) = \dfrac{x^2 - 6x + 4}{(x - 3)^2}$ **17.** $f'(x) = \cos x - x\sin x$

$f'(1) = -\dfrac{1}{4}$ $f'\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{8}(4 - \pi)$

	Function	*Rewrite*	*Differentiate*	*Simplify*
19.	$y = \dfrac{x^2 + 2x}{3}$	$y = \dfrac{1}{3}(x^2 + 2x)$	$y' = \dfrac{1}{3}(2x + 2)$	$y' = \dfrac{2(x + 1)}{3}$
21.	$y = \dfrac{7}{3x^3}$	$y = \dfrac{7}{3}x^{-3}$	$y' = -7x^{-4}$	$y' = -\dfrac{7}{x^4}$
23.	$y = \dfrac{4x^{3/2}}{x}$	$y = 4x^{1/2}$, $x > 0$	$y' = 2x^{-1/2}$	$y' = \dfrac{2}{\sqrt{x}}$, $x > 0$

25. $\dfrac{(x^2 - 1)(-2 - 2x) - (3 - 2x - x^2)(2x)}{(x^2 - 1)^2} = \dfrac{2}{(x + 1)^2}, x \neq 1$

27. $1 - \dfrac{12}{(x + 3)^2} = \dfrac{x^2 + 6x - 3}{(x + 3)^2}$

29. $\dfrac{\sqrt{x}(2) - (2x + 5)\frac{1}{2\sqrt{x}}}{x} = \dfrac{2x - 5}{2x^{3/2}}$

31. $6s^2(s^3 - 2)$ **33.** $-\dfrac{2x^2 - 2x + 3}{x^2(x - 3)^2}$

35. $(3x^3 + 4x)[(x - 5) \cdot 1 + (x + 1) \cdot 1]$
$+ [(x - 5)(x + 1)](9x^2 + 4)$
$= 15x^4 - 48x^3 - 33x^2 - 32x - 20$

37. $\dfrac{(x^2 - c^2)(2x) - (x^2 + c^2)(2x)}{(x^2 - c^2)^2} = -\dfrac{4xc^2}{(x^2 - c^2)^2}$

39. $t(t\cos t + 2\sin t)$ **41.** $-\dfrac{t\sin t + \cos t}{t^2}$

43. $-1 + \sec^2 x = \tan^2 x$ **45.** $\dfrac{1}{4t^{3/4}} + 8\sec t\tan t$

47. $\dfrac{-6\cos^2 x + 6\sin x - 6\sin^2 x}{4\cos^2 x} = \dfrac{3}{2}(-1 + \tan x\sec x - \tan^2 x)$
$= \dfrac{3}{2}\sec x(\tan x - \sec x)$

49. $\csc x\cot x - \cos x = \cos x\cot^2 x$ **51.** $x(x\sec^2 x + 2\tan x)$

2 tan x

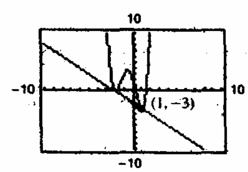
53. $2x\cos x + 2\sin x - x^2\sin x + 2x\cos x$
$= 4x\cos x + (2 - x^2)\sin x$

55. $\left(\dfrac{x + 1}{x + 2}\right)(2) + (2x - 5)\left[\dfrac{(x + 2)(1) - (x + 1)(1)}{(x + 2)^2}\right]$
$= \dfrac{2x^2 + 8x - 1}{(x + 2)^2}$

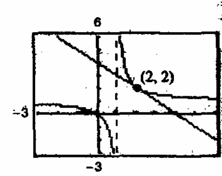
57. $\dfrac{1 - \sin\theta + \theta\cos\theta}{(1 - \sin\theta)^2}$ **59.** $y' = \dfrac{-2\csc x\cot x}{(1 - \csc x)^2}, \quad -4\sqrt{3}$

61. $h'(t) = \dfrac{\sec t(t\tan t - 1)}{t^2}, \quad \dfrac{1}{\pi^2}$

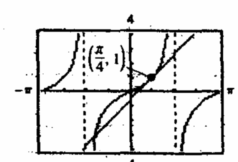
63. (a) $y = -x - 2$
(b)



65. (a) $y = -x + 4$
(b)



67. (a) $4x - 2y - \pi + 2 = 0$
(b)



69. $(0, 0), (2, 4)$ **71.** $f(x) + 2 = g(x)$

73. $n = 1, f'(x) = x\cos x + \sin x$
$n = 2, f'(x) = x^2\cos x + 2x\sin x$
$n = 3, f'(x) = x^3\cos x + 3x^2\sin x$
$n = 4, f'(x) = x^4\cos x + 4x^3\sin x$
$f'(x) = x^n\cos x + nx^{n-1}\sin x$

Sheet #355: Review Rules of Differentiation

3. 2.4a

We conclude this section with a summary of the differentiation rules studied so far. To become skilled at differentiation, you should memorize each rule.

Summary of Differentiation Rules

General Differentiation Rules	Let f, g, and u be differentiable functions of x.	
	Constant Multiple Rule: $\frac{d}{dx}[cf] = cf'$	*Sum or Difference Rule:* $\frac{d}{dx}[f \pm g] = f' \pm g'$
	Product Rule: $\frac{d}{dx}[fg] = fg' + gf'$	*Quotient Rule:* $\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$
Derivatives of Algebraic Functions	*Constant Rule:* $\frac{d}{dx}[c] = 0$	*(Simple) Power Rule:* $\frac{d}{dx}[x^n] = nx^{n-1}$, $\frac{d}{dx}[x] = 1$
Derivatives of Trigonometric Functions	$\frac{d}{dx}[\sin x] = \cos x$ $\frac{d}{dx}[\cos x] = -\sin x$	$\frac{d}{dx}[\tan x] = \sec^2 x$ $\quad \frac{d}{dx}[\sec x] = \sec x \tan x$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad \frac{d}{dx}[\csc x] = -\csc x \cot x$
Chain Rule	*Chain Rule:* $\frac{d}{dx}[f(u)] = f'(u)u'$	*General Power Rule:* $\frac{d}{dx}[u^n] = nu^{n-1}u'$

[redacted] As an aid to memorization, note that the cofunctions (cosine, cotangent, and cosecant) require a negative sign as part of their derivatives.

2.4

EXERCISES FOR SECTION 2.4

Exercises 1–6, complete the table using Example 2 as a model.

	$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1.	$y = (6x - 5)^4$	[redacted]	[redacted]
2.	$y = \frac{1}{\sqrt{x + 1}}$	[redacted]	[redacted]
3.	$y = \sqrt{x^2 - 1}$	[redacted]	[redacted]
4.	$y = 3\tan(\pi x^2)$	[redacted]	[redacted]
5.	$y = \csc^3 x$	[redacted]	[redacted]
6.	$y = \cos\frac{3x}{2}$	[redacted]	[redacted]

Exercises 7–34, find the derivative of the function.

7. $y = (2x - 7)^3$
8. $y = (2x^3 + 1)^2$
9. $g(x) = 3(4 - 9x)^4$
10. $y = 3(4 - x^2)^5$
11. $f(x) = (9 - x^2)^{2/3}$
12. $f(t) = (9t + 2)^{2/3}$
13. $f(t) = \sqrt{1 - t}$
14. $g(x) = \sqrt{5 - 3x}$
15. $y = \sqrt[3]{9x^2 + 4}$
16. $g(x) = \sqrt{x^2 - 2x + 1}$
17. $y = 2\sqrt[4]{4 - x^2}$
18. $f(x) = -3\sqrt[4]{2 - 9x}$
19. $y = \frac{1}{x - 2}$
20. $s(t) = \frac{1}{t^2 + 3t - 1}$
21. $f(t) = \left(\frac{1}{t - 3}\right)^2$
22. $y = -\frac{5}{(t + 3)^3}$
23. $y = \frac{1}{\sqrt{x + 2}}$
24. $g(t) = \sqrt{\frac{1}{t^2 - 2}}$
25. $f(x) = x^2(x - 2)^4$
26. $f(x) = x(3x - 9)^3$
27. $y = x\sqrt{1 - x^2}$
28. $y = \frac{1}{2}x^2\sqrt{16 - x^2}$
29. $y = \frac{x}{\sqrt{x^2 + 1}}$
30. $y = \frac{x}{\sqrt{x^4 + 4}}$
31. $g(x) = \left(\frac{x + 5}{x^2 + 2}\right)^2$
32. $h(t) = \left(\frac{t^2}{t^3 + 2}\right)^2$
33. $f(v) = \left(\frac{1 - 2v}{1 + v}\right)^3$
34. $g(x) = \left(\frac{3x^2 - 2}{2x + 3}\right)^3$

2.4 a 4.

5. $\dfrac{6t+1}{2\sqrt{t}}$ square centimeters per second

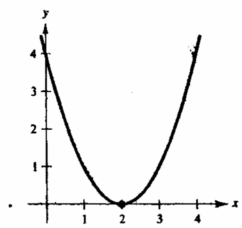
7. (a) $-\$38.13$ (b) $-\$10.37$ (c) $-\$3.80$

The costs decrease with increasing order size.

9. 31.55 bacteria ~~per hour~~ ~~**81.** Proof~~ ~~**83.**~~ $\dfrac{3}{\sqrt{x}}$

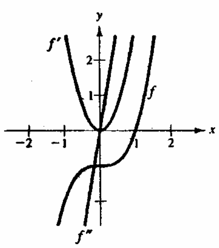
5. $\dfrac{2}{(x-1)^3}$ **87.** $-3\sin x$ **89.** $2x$ **91.** $\dfrac{1}{\sqrt{x}}$

3. Answers will vary. For example: $(x-2)^2$



95. 0 **97.** -10

99.



01. $v(3) = 27$ meters per second

$a(3) = -6$ meters per second per second

The speed of the object is decreasing, but the rate of that decrease is increasing.

03. (a) 2.4 ft/sec^2 (b) 1.2 ft/sec^2 (c) 0.5 ft/sec^2

105. (a) $f''(x) = g(x)h''(x) + 2g'(x)h'(x) + g''(x)h(x)$

$f'''(x) = g(x)h'''(x) + 3g'(x)h''(x) + 3g''(x)h'(x) + g'''(x)h(x)$

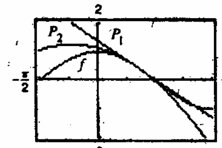
$f^{(4)}(x) = g(x)h^{(4)}(x) + 4g'(x)h'''(x) + 6g''(x)h''(x) + 4g'''(x)h'(x) + g^{(4)}(x)h(x)$

(b) $f^{(n)}(x) = g(x)h^{(n)}(x) + \dfrac{n!}{1!(n-1)!}g'(x)h^{(n-1)}(x) + \dfrac{n!}{2!(n-2)!}g''(x)h^{(n-2)}(x) + \cdots + \dfrac{n!}{(n-1)!1!}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)$

107. (a) $P_1(x) = -\dfrac{\sqrt{3}}{2}\left(x - \dfrac{\pi}{3}\right) + \dfrac{1}{2}$

$P_2(x) = -\dfrac{1}{4}\left(x - \dfrac{\pi}{3}\right)^2 - \dfrac{\sqrt{3}}{2}\left(x - \dfrac{\pi}{3}\right) + \dfrac{1}{2}$

(b)



(c) P_2

(d) P_1 and P_2 become less accurate as you move farther from $x = a$.

109. False: $dy/dx = f(x)g'(x) + g(x)f'(x)$ **111.** True

113. True **115.** $f'(x) = 2|x|$; $f''(0)$ does not exist.

~~Section 2.4 (page 133)~~

	$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1.	$y = (6x-5)^4$	$u = 6x-5$	$y = u^4$
3.	$y = \sqrt{x^2-1}$	$u = x^2-1$	$y = \sqrt{u}$
5.	$y = \csc^3 x$	$u = \csc x$	$y = u^3$

7. $6(2x-7)^2$ **9.** $-108(4-9x)^3$

11. $\dfrac{2}{3}(9-x^2)^{-1/3}(-2x) = -\dfrac{4x}{3(9-x^2)^{1/3}}$

13. $\dfrac{1}{2}(1-t)^{-1/2}(-1) = -\dfrac{1}{2\sqrt{1-t}}$

15. $\dfrac{1}{3}(9x^2+4)^{-2/3}(18x) = \dfrac{6x}{(9x^2+4)^{2/3}}$

17. $\dfrac{1}{2}(4-x^2)^{-3/4}(-2x) = \dfrac{-x}{\sqrt[4]{(4-x^2)^3}}$

19. $-\dfrac{1}{(x-2)^2}$ **21.** $-2(t-3)^{-3}(1) = -\dfrac{2}{(t-3)^3}$

23. $-\dfrac{1}{2(x+2)^{3/2}}$

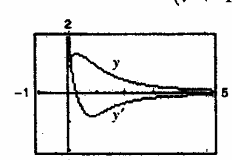
25. $x^2[4(x-2)^3(1)] + (x-2)^4(2x) = 2x(x-2)^3(3x-2)$

27. $x\left(\dfrac{1}{2}\right)(1-x^2)^{-1/2}(-2x) + (1-x^2)^{1/2}(1) = \dfrac{1-2x^2}{\sqrt{1-x^2}}$

29. $\dfrac{(x^2+1)^{1/2}(1) - x(1/2)(x^2+1)^{-1/2}(2x)}{x^2+1} = \dfrac{1}{(x^2+1)^{3/2}}$

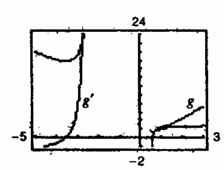
31. $\dfrac{-2(x+5)(x^2+10x-2)}{(x^2+2)^3}$ **33.** $\dfrac{-9(2v-1)^2}{(v+1)^4}$

35. $\dfrac{1-3x^2-4x^{3/2}}{2\sqrt{x}(x^2+1)^2}$



The zero of y' corresponds to the point on the graph of the function where the tangent line is horizontal.

37. $\dfrac{3t(t^2+3t-2)}{(t^2+2t-1)^{3/2}}$



The zeros of $g'(t)$ correspond to the points on the graph of the function where the tangent line is horizontal.

Sheet #355: Review Rules of Differentiation

2.4 b (5-)

In Exercises 35–44, use a computer algebra system to find the derivative of the function. Then use the utility to graph the function and its derivative on the same set of coordinate axes. Describe the behavior of the function that corresponds to any zeros of the graph of the derivative.

35. $y = \dfrac{\sqrt{x}+1}{x^2+1}$

36. $y = \sqrt{\dfrac{2x}{x+1}}$

37. $g(t) = \dfrac{3t^2}{\sqrt{t^2+2t-1}}$

38. $f(x) = \sqrt{x}(2-x)^2$

39. $y = \sqrt{\dfrac{x+1}{x}}$

40. $y = (t^2-9)\sqrt{t+2}$

41. $s(t) = \dfrac{-2(2-t)\sqrt{1+t}}{3}$

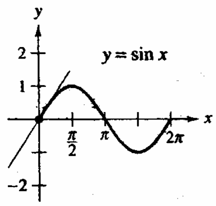
42. $g(x) = \sqrt{x-1} + \sqrt{x+1}$

43. $y = \dfrac{\cos \pi x + 1}{x}$

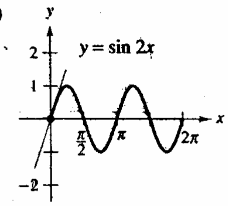
44. $y = x^2 \tan \dfrac{1}{x}$

In Exercises 45 and 46, find the slope of the tangent line to the sine function at the origin. Compare this value with the number of complete cycles in the interval $[0, 2\pi]$. What can you conclude about the slope of the sine function $\sin ax$ at the origin?

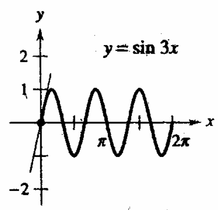
45. (a)



(b)



46. (a)



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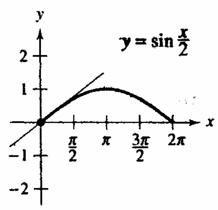


In Exercises 47–66, find the derivative of the function.

47. $y = \cos 3x$

48. $y = \sin \pi x$

49. $g(x) = 3 \tan 4x$

50. $h(x) = \sec x^2$

51. $y = \sin(\pi x)^2$

52. $y = \cos(1-2x)^2$

53. $h(x) = \sin 2x \cos 2x$

54. $g(\theta) = \sec(\frac{1}{2}\theta)\tan(\frac{1}{2}\theta)$

55. $f(x) = \dfrac{\cot x}{\sin x}$

56. $g(v) = \dfrac{\cos v}{\csc v}$

57. $y = 4\sec^2 x$

58. $y = 2\tan^3 x$

59. $f(\theta) = \frac{1}{4}\sin^2 2\theta$

60. $g(t) = 5\cos^2 \pi t$

61. $f(t) = 3\sec^2(\pi t - 1)$

62. $h(t) = 2\cot^2(\pi t + 2)$

63. $y = \sqrt{x} + \frac{1}{4}\sin(2x)^2$

64. $y = 3x - 5\cos(\pi x)^2$

65. $y = \sin(\cos x)$

66. $y = \sin\sqrt[3]{x} + \sqrt[3]{\sin x}$

In Exercises 67–74, evaluate the derivative of the function at the indicated point. Use a graphing utility to verify your result.

	Function	*Point*
67.	$s(t) = \sqrt{t^2+2t+8}$	$(2, 4)$
68.	$y = \sqrt[5]{3x^3+4x}$	$(2, 2)$
69.	$f(x) = \dfrac{3}{x^3-4}$	$\left(-1, -\frac{3}{5}\right)$
70.	$f(x) = \dfrac{1}{(x^2-3x)^2}$	$\left(4, \frac{1}{16}\right)$
71.	$f(t) = \dfrac{3t+2}{t-1}$	$(0, -2)$
72.	$f(x) = \dfrac{x+1}{2x-3}$	$(2, 3)$
73.	$y = 37 - \sec^3(2x)$	$(0, 36)$
74.	$y = \dfrac{1}{x} + \sqrt{\cos x}$	$\left(\frac{\pi}{2}, \frac{2}{\pi}\right)$

In Exercises 75–78, (a) find an equation of the tangent line to the graph of f at the indicated point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of a graphing utility to confirm your results.

	Function	*Point*
75.	$f(x) = \sqrt{3x^2-2}$	$(3, 5)$
76.	$f(x) = \frac{1}{3}x\sqrt{x^2+5}$	$(2, 2)$
77a 77.	$f(x) = \sin 2x$ PART a ONLY	$(\pi, 0)$
78.	$f(x) = \tan^2 x$	$\left(\frac{\pi}{4}, 1\right)$

In Exercises 79–82, find the second derivative of the function.

79. $f(x) = 2(x^2-1)^3$

80. $f(x) = \dfrac{1}{x-2}$

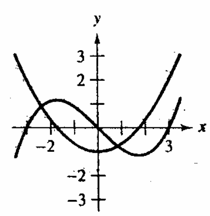
81. $f(x) = \sin x^2$

82. $f(x) = \sec^2 \pi x$

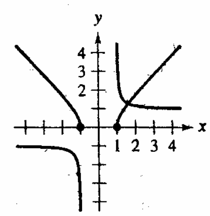
Getting at the Concept

In Exercises 83–86, the graphs of a function f and its derivative f' are shown. Label the graphs as f or f' and write a short paragraph stating the criteria used in making the selection. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

83.

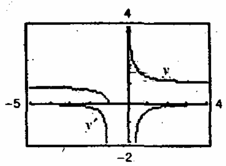


84.



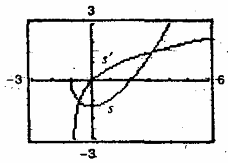
2.4 b 6.

39. $-\dfrac{\sqrt{\dfrac{x+1}{x}}}{2x(x+1)}$



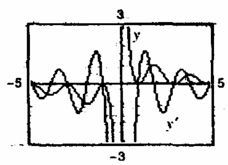
y' has no zeros.

41. $\dfrac{t}{\sqrt{1+t}}$



The zero of $s'(t)$ corresponds to the point on the graph of the function where the tangent line is horizontal.

43. $-\dfrac{\pi x \sin \pi x + \cos \pi x + 1}{x^2}$



The zeros of y' correspond to the points on the graph of the function where the tangent lines are horizontal.

45. (a) 1 (b) 2; The slope of $\sin ax$ at the origin is a.

47. $-3\sin 3x$ **49.** $12\sec^2 4x$ **51.** $2\pi^2 x \cos(\pi x)^2$

53. $2\cos(4x)$ **55.** $\dfrac{-1-\cos^2 x}{\sin^3 x}$

57. $8\sec^2 x \tan x = \dfrac{8\sin x}{\cos^3 x}$ **59.** $\sin 2\theta \cos 2\theta = \dfrac{1}{2}\sin 4\theta$

61. $\dfrac{6\pi \sin(\pi t - 1)}{\cos^3(\pi t - 1)}$ **63.** $\dfrac{1}{2\sqrt{x}} + 2x\cos(2x)^2$

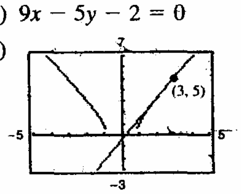
65. $-\sin x \cos(\cos x)$ **67.** $s'(t) = \dfrac{t+1}{\sqrt{t^2+2t+8}}, \dfrac{3}{4}$

69. $f'(x) = \dfrac{-9x^2}{(x^3-4)^2}, -\dfrac{9}{25}$ **71.** $f'(t) = \dfrac{-5}{(t-1)^2}, -5$

73. $y' = -6\sec^3(2x)\tan(2x), 0$

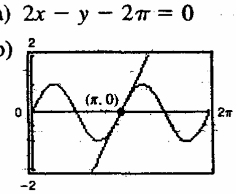
75. (a) $9x - 5y - 2 = 0$

(b)



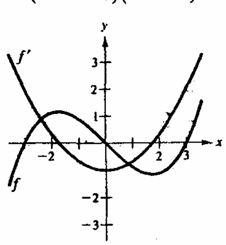
77. (a) $2x - y - 2\pi = 0$

(b)



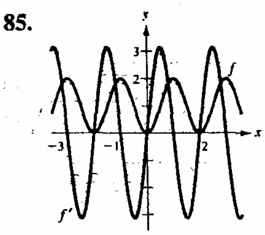
79. $12(5x^2-1)(x^2-1)$ **81.** $2(\cos x^2 - 2x^2 \sin x^2)$

83.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

85.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

87. The rate of change of g will be three times as fast as the rate of change of f.

89. (a) 24 (b) Not possible because $g'(h(5))$ is not known.
(c) $\frac{4}{3}$ (d) 162

91. (a) 1.461 (b) -1.016

93. 0.2 radian, 1.45 radians per second **95.** 0.04224

97. (a) $x = -1.637t^3 + 19.31t^2 - 0.5t - 1$

(b) $\dfrac{dC}{dt} = -294.66t^2 + 2317.2t - 30$

(c) Because x, the number of units produced in t hours, is not a linear function, and therefore the cost with respect to time t is not linear.

not

time

99. (a) $f'(x) = \beta \cos \beta x$

$f''(x) = -\beta^2 \sin \beta x$

$f'''(x) = -\beta^3 \cos \beta x$

$f^{(4)}(x) = \beta^4 \sin \beta x$

(b) $f''(x) + \beta^2 f(x) = -\beta^2 \sin \beta x + \beta^2 \sin \beta x = 0$

(c) $f^{(2k)}(x) = (-1)^k \beta^{2k} \sin \beta x$

$f^{(2k-1)}(x) = (-1)^{k+1} \beta^{2k-1} \cos \beta x$

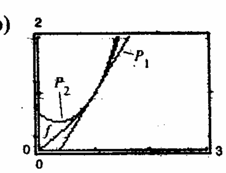
101. (a) 0 (b) $\dfrac{5}{8}$ **103.** Proof **105.** $\dfrac{2(2x-3)}{|2x-3|}, x \neq \dfrac{3}{2}$

107. $-|x|\sin x + \dfrac{x}{|x|}\cos x, \quad x \neq 0$

109. (a) $P_1(x) = \dfrac{\pi}{2}(x-1) + 1$

$P_2(x) = \dfrac{\pi^2}{8}(x-1)^2 + \dfrac{\pi}{2}(x-1) + 1$

(b)



(c) P_2 accurately

(d) P_1 and P_2 become less accurate as you move farther from $x = 1$.

from

111. False. $y' = -\frac{1}{2}(1-x)^{-1/2}$ **113.** True

Section 2.5 (page 142)

1. $-\dfrac{x}{y}$ **3.** $-\sqrt{\dfrac{y}{x}}$ **5.** $\dfrac{y-3x^2}{2y-x}$ **7.** $\dfrac{1-3x^2y^3}{3x^3y^2-1}$

9. $\dfrac{6xy-3x^2-2y^2}{4xy-3x^2}$ **11.** $\dfrac{\cos x}{4\sin 2y}$

Sheet #355: Review Rules of Differentiation

5.4 7

In Exercises 39–58, find the derivative of the function.

39. $f(x) = e^{2x}$
40. $f(x) = e^{1-x}$
41. $y = e^{-2x+x^2}$
42. $y = e^{-x^2}$
43. $y = e^{\sqrt{x}}$
44. $y = x^2e^{-x}$
45. $g(t) = (e^{-t} + e^{t})^3$
46. $g(t) = e^{-3/t^2}$
47. $y = \ln(e^{x^2})$
48. $y = \ln\left(\frac{1 + e^x}{1 - e^x}\right)$
49. $y = \ln(1 + e^{2x})$
50. $y = \ln\frac{e^x + e^{-x}}{2}$
51. $y = \frac{2}{e^x + e^{-x}}$
52. $y = \frac{e^x - e^{-x}}{2}$
53. $y = x^2e^x - 2xe^x + 2e^x$
54. $y = xe^x - e^x$
55. $f(x) = e^{-x}\ln x$
56. $f(x) = e^3\ln x$
57. $y = e^x(\sin x + \cos x)$
58. $y = \ln e^x$

In Exercises 59 and 60, use implicit differentiation to find dy/dx.

59. $xe^y - 10x + 3y = 0$
60. $e^{xy} + x^2 - y^2 = 10$

In Exercises 61 and 62, find the second derivative of the function.

61. $f(x) = (3 + 2x)e^{-3x}$
62. $g(x) = \sqrt{x} + e^x\ln x$

In Exercises 63 and 64, show that the function $y = f(x)$ is a solution of the differential equation.

63. $y = e^x(\cos\sqrt{2}x + \sin\sqrt{2}x)$
$y'' - 2y' + 3y = 0$
64. $y = e^x(3\cos 2x - 4\sin 2x)$
$y'' - 2y' + 5y = 0$

In Exercises 65–72, find the extrema and the points of inflection (if any exist) of the function. Use a graphing utility to graph the function and confirm your results.

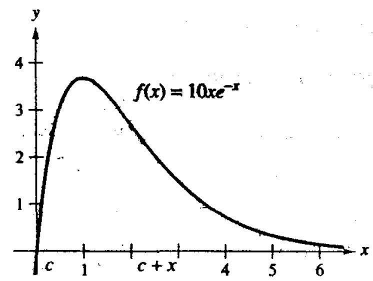
65. $f(x) = \frac{e^x + e^{-x}}{2}$
66. $f(x) = \frac{e^x - e^{-x}}{2}$
67. $g(x) = \frac{1}{\sqrt{2\pi}}e^{-(x-2)^2/2}$
68. $g(x) = \frac{1}{\sqrt{2\pi}}e^{-(x-3)^2/2}$
69. $f(x) = x^2e^{-x}$
70. $f(x) = xe^{-x}$
71. $g(t) = 1 + (2 + t)e^{-t}$
72. $f(x) = -2 + e^{3x}(4 - 2x)$

73. ***Area*** Find the area of the largest rectangle that can be inscribed under the curve $y = e^{-x^2}$ in the first and second quadrants.

74. ***Area*** Perform the following steps to find the maximum a of the rectangle shown in the figure.
(a) Solve for c in the equation $f(c) = f(c + x)$.
(b) Use the result in part (a) to write the area A as a functio x. [*Hint:* $A = xf(c)$]
(c) Use a graphing utility to graph the area function. Use graph to approximate the dimensions of the rectangl maximum area. Determine the maximum area.
(d) Use a graphing utility to graph the expression for c fou in part (a). Use the graph to approximate

$$\lim_{x\to 0^+} c \quad \text{and} \quad \lim_{x\to\infty} c.$$

Use this result to describe the changes in dimensions a position of the rectangle for $0 < x < \infty$.



75. Verify that the function

$$y = \frac{L}{1 + ae^{-x/b}}, \quad a > 0, b > 0, L > 0$$

increases at a maximum rate when $y = L/2$.

76. Find the point on the graph of $y = e^{-x}$ where the normal line the curve passes through the origin. (Use Newton's Method the root-finding capabilities of a graphing utility.)

77. Find, to three decimal places, the value of x such that

$e^{-x} = x$.

(Use Newton's Method or the root-finding capabilities of graphing utility.)

78. ***Depreciation*** The value V of an item t years after it purchased is

$V = 15{,}000e^{-0.6286t}, \quad 0 \le t \le 10.$

(a) Use a graphing utility to graph the function.
(b) Find the rate of change of V with respect to t when $t =$ and $t = 5$.
(c) Use a graphing utility to graph the tangent line to the fun tion when $t = 1$ and $t = 5$.

79. ***Writing*** Consider the function

$$f(x) = \frac{2}{1 + e^{1/x}}.$$

(a) Use a graphing utility to graph f.
(b) Write a short paragraph explaining why the graph has horizontal asymptote at $y = 1$ and why the function has nonremovable discontinuity at $x = 0$.

Sheet #355: Review Rules of Differentiation

5.4

8.

81. $-\frac{1}{11}$ **83.** 32 **85.** 600 **87.** $(g^{-1} \circ f^{-1})(x) = \dfrac{x+1}{2}$

89. $(f \circ g)^{-1}(x) = \dfrac{x+1}{2}$

91. Let $y = f(x)$ be one-to-one. Solve for x as a function of y. Interchange x and y to get $y = f^{-1}(x)$. Let the domain of f^{-1} be the range of f. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

Example: $f(x) = x^3$

$$y = x^3$$
$$x = \sqrt[3]{y}$$
$$y = \sqrt[3]{x}$$
$$f^{-1}(x) = \sqrt[3]{x}$$

93. Answers will vary. Example: $y = x^4 - 2x^3$

95. Many x-values yield the same y-value.

For example, $f(\pi) = 0 = f(0)$.

The graph is not continuous at $x = \dfrac{(2n-1)\pi}{2}$, where n is an integer.

97. Proof **99.** Proof

101. False. Let $f(x) = x^2$. **103.** True

105. No. Let $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 1 - x, & 1x \le 2 \end{cases}$. **107.** $\sqrt{17}$

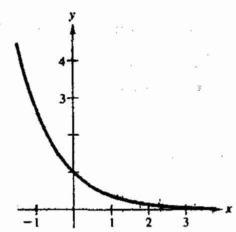
Section 5.4 (page 347)

1. $\ln 1 = 0$ **3.** $e^{0.6931\ldots} = 2$ **5.** $x = 4$

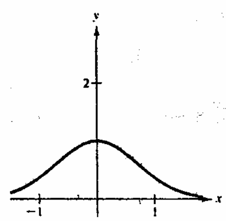
7. $x \approx 2.485$ **9.** $x = 0$ **11.** $x \approx 0.511$

13. $x \approx 7.389$ **15.** $x \approx 10.389$ **17.** $x \approx 5.389$

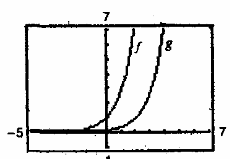
19.



21.

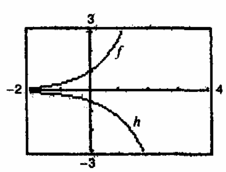


23. (a)



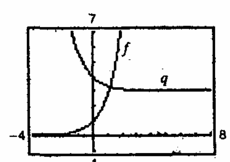
Translation 2 units to the right

(b)



Reflection in the x-axis and a vertical shrink

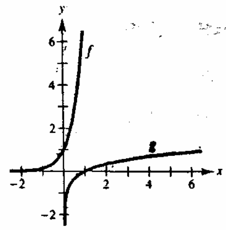
(c)



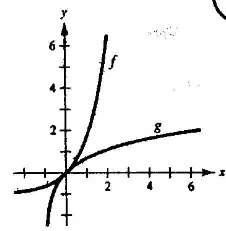
Reflection in the y-axis and a translation 3 units upward

25. c **26.** d **27.** a **28.** b

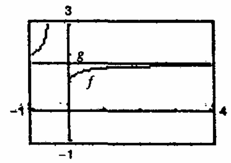
29.



31.



33.



$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = e^{0.5}$

35. $2.7182805 < e$ **37.** (a) 3 (b) -3

39. $2e^{2x}$ **41.** $2(x-1)e^{-2x+x^2}$ **43.** $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

45. $3(e^{-t} + e^{t})^2(e^{t} - e^{-t})$ **47.** $2x$ **49.** $\dfrac{2e^{2x}}{1+e^{2x}}$

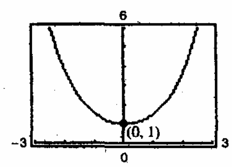
51. $\dfrac{-2(e^x - e^{-x})}{(e^x + e^{-x})^2}$ **53.** x^2e^x **55.** $e^{-x}\left(\dfrac{1}{x} - \ln x\right)$

57. $2e^x \cos x$ **59.** $\dfrac{10 - e^y}{xe^y + 3}$ **61.** $3(6x+5)e^{-3x}$

63. $y'' - 2y' + 3y = 0$

$$e^x\left[-\cos\sqrt{2}x - \sin\sqrt{2}x - 2\sqrt{2}\sin\sqrt{2}x + 2\sqrt{2}\cos\sqrt{2}x\right] -$$
$$2e^x\left[-\sqrt{2}\sin\sqrt{2}x + \sqrt{2}\cos\sqrt{2}x + \cos\sqrt{2}x + \sin\sqrt{2}x\right] +$$
$$3e^x\left[\cos\sqrt{2}x + \sin\sqrt{2}x\right]$$
$$= 0$$
$$0 = 0$$

65. Relative minimum: $(0, 1)$



67. Relative maximum: $\left(2, \dfrac{1}{\sqrt{2\pi}}\right)$

Points of inflection: $\left(1, \dfrac{e^{-0.5}}{\sqrt{2\pi}}\right), \left(3, \dfrac{e^{-0.5}}{\sqrt{2\pi}}\right)$