Sheet #265 - Ch. 1 & 2 Review

Name _____ Per/Sec. _____

No calculator allowed. Be prepared to do any question without multiple choices being provided. Show your work and/or give brief explanations where relevant. It is important for you to do the questions at the end!

1. If $f(x) = \begin{cases} 5 & \text{for } x = 2, \\ 3x+1 & \text{for } x \neq 2 \end{cases}$ then $\lim_{x\to 2} f(x) =$ _____.

2. Find A so that $\lim_{x\to 4} \dfrac{x^2 + Ax + 20}{x-4}$ exists.

3. $\lim_{x\to\infty} \dfrac{3x^4 - 5x^3 + 709}{7x^4 + 9x^2 + 11}$ is

A) $-\frac{3}{7}$ B) $\frac{3}{7}$ C) 0 D) ∞ E) 3

4. $\lim_{x\to -4} \dfrac{x^2 - 3x - 28}{(x+4)^2} =$

A) 1 B) ∞ C) undefined
D) $\dfrac{1}{4}$ E) $\dfrac{1}{16}$

5. $\lim_{x\to 0^+} \dfrac{3x+5}{x}$ is

A) $-\infty$ B) 0 C) 2^- D) 2^+ E) ∞

6. $\lim_{x\to -\infty} \dfrac{x}{(x-5)(x+7)}$ is

A) 1 B) $-\infty$ C) 0 D) $-\dfrac{5}{7}$ E) ∞

7. $\lim_{x\to\infty} \dfrac{\sqrt{5x^2+3x}}{x} =$ _____

8. $\lim_{x\to -\infty} \dfrac{\sqrt{13x^2+5x}}{x} =$ _____

9. Given a function defined by $f(x) = \dfrac{2x+2}{x^2+5x+4}$, for what value(s) of x is the function discontinuous?

A) 1 only B) 2 C) 1, 4
D) -4 only E) -4, -1

10. Which of the following functions are continuous for all real numbers x?

I. $y = \dfrac{x^2 - 13}{x^2 + 7}$

II. $y = -\dfrac{5}{x^2}$

III. $y = |3x + 11|$

A) I and III only B) I only
C) III only D) I and II only
E) II and III only

11. Let f be defined as follows:

$$f(x) = \begin{cases} \dfrac{x^2 - c^2}{x - c} & \text{for } x \neq c, \\ c^2 & \text{for } x = c \end{cases}$$

Which of the following are true about f?

I. $\lim_{x \to c} f(x)$ exists

II. $f(c)$ exists

III. $f(x)$ is continuous at $x = c$

A) None B) I only

C) II only D) I and II only

E) I, II, and III

12. Consider the function

$$f(x) = \begin{cases} x^2 & \text{for } -1 < x < 1, \\ 1 & \text{for } x > 1, \\ 1 & \text{for } x \leq -1 \end{cases}$$

At $x = -1$ the function has

A) a jump discontinuity

B) an infinite discontinuity

C) a removable discontinuity

D) a point at which the function is continuous

E) a point at which the function is differentiable

13. Consider $f(x) = \begin{cases} x^2 - 5 & \text{for } x < 0, \\ 3 & \text{for } x = 0, \\ x^2 + 5 & \text{for } x > 0 \end{cases}$

a) $\lim_{x \to 0^+} f(x) =$ _____

b) $\lim_{x \to 0^-} f(x) =$ _____

c) $\lim_{x \to 3} f(x) =$ _____

d) Where is $f(x)$ discontinuous? _____

e) If a function is continuous at $x = a$, does this necessarily mean that $\lim_{x \to a}$ exists? _____

14. Holes are removable discontinuities. Consider $f(x) = \begin{cases} x + c & \text{for } x < -2, \\ cx^2 + 7 & \text{for } x \geq -2 \end{cases}$

For what value of the constant c is f continuous for all real numbers?

15. What is $\lim_{h \to 0} \dfrac{\sqrt{25 + h} - \sqrt{25}}{h}$?

A) $\dfrac{1}{50}$ B) $\dfrac{1}{10}$ C) 10

D) 5 E) $\dfrac{1}{2\sqrt{25 + h}}$

16. A function f is given by the table shown.

Estimate $f'(5.5)$:

x	3.7	4.3	4.9	5.5	6.1
$f(x)$	1.8	3.4	4.6	6.4	8.4

A) 0.316 B) 3.167 C) 0.300

D) 6.400 E) 0.297

17. The table shows the position of an object moving along a line at 6 second intervals.

Estimate the velocity, in units/sec, at $t = 21$.

$t(sec)$	0	6	12	18	24
position	40	38	35	30	18

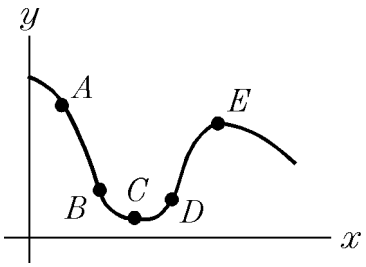
A) 0.500 B) -2.000 C) -0.500

D) 2.000 E) -1.000

18. If $y = f(x)$, then $dy/dx = f'(x)$.

At which of the five points shown on the graph is $\frac{dy}{dx}$ negative? Choose the *best* answer.

A) A and B
B) B only
C) C only
D) C, D, and E
E) D only

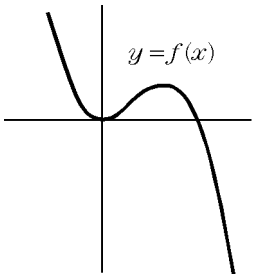


19. If $y = f(x)$, then $d^2y/dx^2 = f''(x)$.

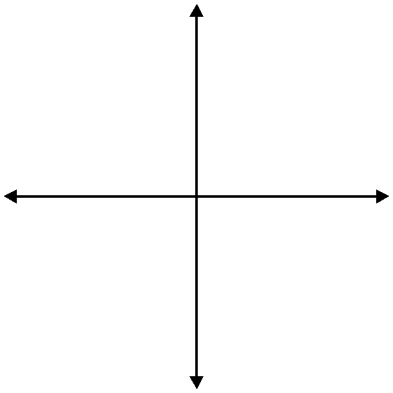
At which of the five points shown on the graph is $\frac{d^2y}{dx^2}$ negative? Choose the *best* answer.

A) A and E
B) B and D
C) C only
D) B, C, and D
E) E only

20.



Given the graph of $y = f(x)$, sketch the graph of $f'(x)$



21. The position of an object is given by $s = t^2 + 5t - 30$. What is its average velocity for $2 \le t \le 4$?

A) 5 B) -11 C) 11 D) -5 E) 5.5

22. The volume of cubic feet of water in a pool is given by the equation $V(t) = 2(t+3)^2$ for $t \ge 0$ and t is in seconds.

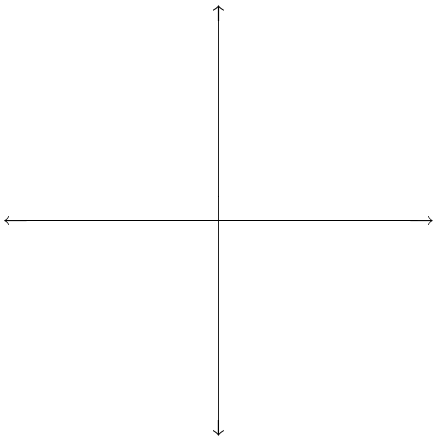
a) What is the average rate of change from $t = 4$ to $t = 6$?

b) What is the average rate of change over the time interval $[4, 4 + \Delta t]$?

c) What is the instantaneous rate of change at $t = 3$?

d) What is the instantaneous rate of change at any time t?

23.



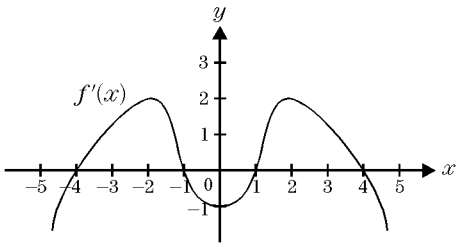
Given a continuous function f and the following information:

Interval	Sign of f'	Sign of f''
$x < 1$	$-$	$+$
$1 < x < 5$	$+$	$+$
$5 < x < 7$	$+$	$-$
$7 < x$	$-$	$-$

Sketch a possible graph of f.

24. The graph of $f'(x)$ is given. The question is about the graph of $f(x)$.

The graph $f(x)$ has horizontal tangents when $x =$



A) $-2,\ 0,\ 2$ B) $-2,\ 2$

C) $-1,\ 1$ D) $-1,\ 2$

E) $-4,\ -1,\ 1,\ 4$

25. Find an equation of the tangent line to the curve $f(x) = x^2 - 10$ passing through the point $(5, 1)$.

A) $y - 1 = -10(x - 5)$ B) $y + 5 = -10(x + 1)$

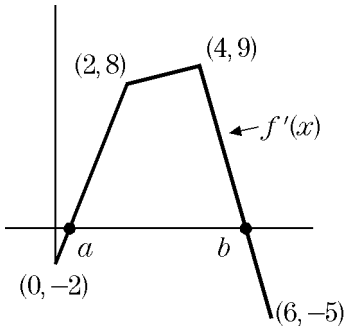
C) $y + 1 = 10(x + 5)$ D) $y - 1 = 10(x - 5)$

E) $y - 5 = 10(x - 1)$

26. At what point is the slope of $f(x) = -3x^2 + 5x + 10$ equal to 17?

27. The graph shows the velocity of an object that is moving along a straight line for t on $[0, 6]$.

When is the speed of the object the least?



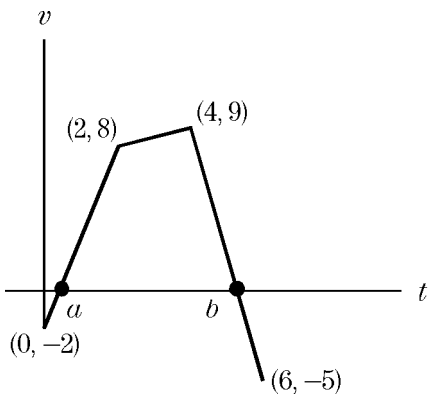
A) at $t = 0$ B) at $t = 6$

C) at $t = a$ and $t = b$ D) at $t = 2$

E) at $t = -5$

28. The graph shows the velocity of an object that is moving along a straight line for t on $[0, 6]$.

When is the acceleration of the object negative?



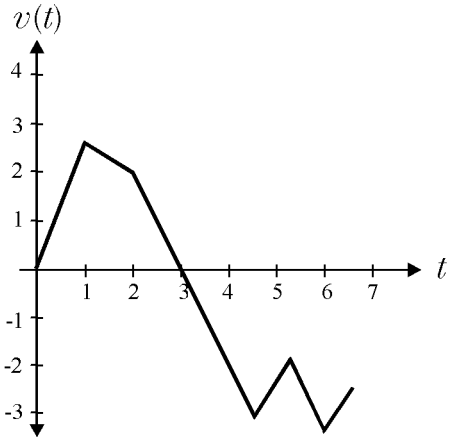
A) (a, b) B) $(b, 6)$

C) $(0, 2)$ and $(2, 4)$ D) $(4, b)$

E) $(4, 6)$

29. The graph shows the velocity of an object that is moving along a straight line for t on $[0, 7]$.

At what time(s) t does the object reverse direction?



A) 3 and 5 B) 4, 5, and 6
C) 1, 4, 5, and 6 D) 1 only
E) 3 only

30. The graph shows the velocity of an object that is moving along a straight line for t on $[0, 6]$.

The object is furthest to the right when $t =$ _____.

A) 1 B) 3 C) 4 D) 6 E) 7

31. A mouse is running through a straight pipe. The velocity, $v(t)$, of the mouse is given at time t for $0 \le t \le 7$.

According to the graph, at what time t is the mouse's speed the greatest?

A) 1 B) 3 C) 4 D) 5 E) 6

32. A particle starts at time $t = 0$ and moves along the x-axis so that its position at any time $t \ge 0$ is $x(t) = t^3 - \frac{21}{2}t^2 + 30t - 8$. During what time intervals is the particle moving to the left?

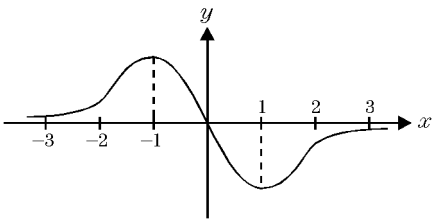
A) $0 < t < 2$ B) $1 < t < 5$ C) $2 < t < 5$
D) $t < 1$ E) $t > 5$

33. A small rocket is projected vertically upward with an initial velocity of 128 ft/sec and moves according to the law $s = 128t - 16t^2$. How many seconds will it be before it reaches it maximum height?

A) 6 B) 32 C) 4 D) 112 E) 8

34. The figure shows the graph of f', the derivative of the function f. The domain of the function f is $-10 \le x \le 10$.

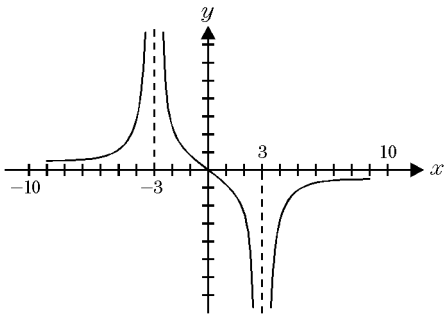
For what value(s) is the graph of f concave downwards?



A) $-1 < x < 1$ B) $-3 < x < 3$ C) $-3 < x < 0$
D) $0 < x < 3$ E) $\emptyset$

35. The figure shows the graph of f', the derivative of the function f. The domain of the function f is $-10 \leq x \leq 10$.

For what value(s) is the graph of f concave downwards?



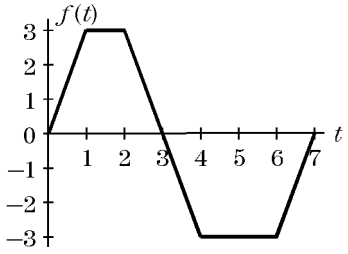
A) $10 < x < -3$ only

B) $3 < x < 10$ only

C) $-3 < x < 3$ only

D) $10 < x < -3$ and $3 < x < 10$

E) $-3 < x < 0$ and $3 < x < 10$

36. 

The graph shows the velocity $v = f(t)$ in m/sec of a particle versus elapsed time t in sec. Give units in your answers.

a) At what time(s) does the particle change direction?

b) What is the largest value for the speed?

c) What is the most negative value of the acceleration? (Only consider times for which the acceleration is well-defined.)

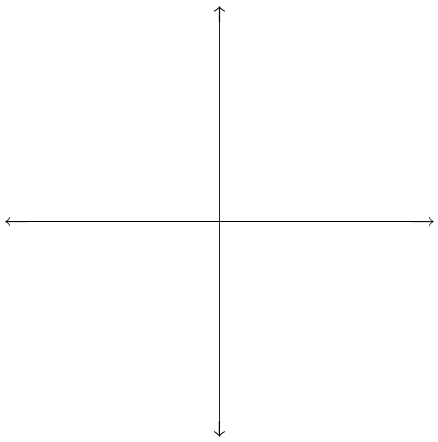
d) What is the net change in position (displacement) from $t = 1$ sec to $t = 2$ sec?

e) What is the largest value for the position?

f) What is the net change in position from $t = 0$ sec to $t = 7$ sec?

37. Sketch a possible graph of $y = f(x)$ if $f(x)$ has the following properties:

- $f(x)$ is continuous everywhere except at $x = 4$.
- $f(x)$ is differentiable everywhere except at $x = 2$ and $x = 4$.
- $f''(x) > 0$ everywhere that $f'(x)$ is defined.



38. Let $f(t)$ be the number of centimeters of rainfall that has fallen since midnight, where t is the number of hours. Interpret the following in practical terms, giving units.

a) $f(10) = 3.1$

b) $f^{-1}(5) = 16$

c) $f'(10) = 0.4$

d) $(f^{-1})'(5) = 2$

e) Using the above values, give the best estimate for how much rain will have fallen by noon.

(Based on CSV textbook Section 2.4, p. 92, question 15.)

39. Let $g(v)$ is the fuel efficiency in miles per gallon of a car going at v miles per hour.

a) What are the units of $g'(55)$?

b) What is the practical meaning of the statement $g'(55) = -0.54$?

c) If $g(55) = 40$, using the information above, what is the best estimate for the fuel efficiency when the car is traveling at 65 miles per hour?

(Based on CSV textbook Section 2.4, p. 92, question 17.)

40. A continuous function defined for all x has the following properties:

- $f(x)$ is increasing.
- $f(x)$ is concave down.
- $f(5) = 2$
- $f'(5) = 0.5$

a) Sketch a possible graph of $f(x)$.

b) How many zeros does $f(x)$ have?

c) What can you say about the locations of the zeros?

d) What is $\lim_{x \to -\infty} f(x)$?

e) Is it possible that $f'(1) = 1$? Why or why not?

f) Is it possible that $f'(1) = 0.25$? Why or why not?

(From CSV textbook Section 2.5, p. 98, question 23.)