SHEET #221:

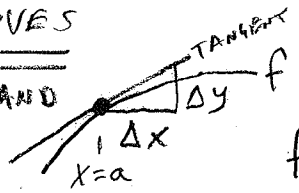
METHODS FOR FINDING DERIVATIVES

APPROXIMATIONS ..... EXACT

GRAPHICAL

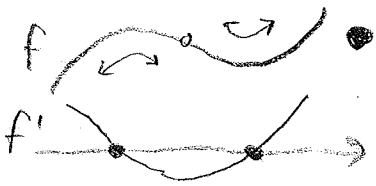
AT A POINT

- DRAW TANGENT LINE AND ESTIMATE $\Delta y/\Delta x$



$$f'(a) \approx \frac{\Delta y}{\Delta x}$$

FUNCTIONS



- LOOK FOR SLOPE = 0, POSITIVE SLOPE, NEGATIVE SLOPE, INCREASING & DECREASING SLOPE, MAXIMA, MINIMA, POINTS OF INFLECTION

- BUILD A TABLE OF POINTS

NUMERICAL

AT A POINT

x	2	5	8	11
f	7	10	16	28

- CALCULATE $\Delta y/\Delta x$ (AVERAGE RATE OF CHANGE) USING DIFFERENCE QUOTIENTS

DIFF. QUOT. $f'(a) \approx \frac{f(a+H)-f(a)}{H}$. $f'(5) \approx \frac{f(8)-f(5)}{8-5} = \frac{16-10}{8-5} = \frac{6}{3} = 2.$

SYMMETRIC DIFF. QUOT $f'(a) \approx \frac{f(a+H)-f(a-H)}{2H}$. $f'(5) \approx \frac{f(8)-f(2)}{8-2} = \frac{16-7}{8-2} = \frac{9}{6} = 1.5$

- USE CALCULATOR TO EVALUATE THE SYMMETRIC DIFF. QUOT.

2nd CALC 6: dy/dx. MATH 8: $nDeriv(Y_1, X, a, H)$ DEFAULT $H = 0.001$

$d(e^x)/dx|_{x=1} \approx nDeriv(e^x, X, 1) = 2.718282282 (\approx e)$

FUNCTION $Y_2 = nDeriv(Y_1, X, X)$ GRAPH

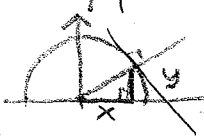
AT A POINT $Y_1 = \cos(X)$

$(Y_1(\pi+H)-Y_1(\pi))/H$	-0.998	-0.99998	-0.9999998 ↓
H	0.1	0.01	0.001

- LIMIT TRENDS AS $H \to 0$ $y'(\pi/2) = -1$ from table.

ANALYTICAL = GEOMETRIC

FUNCTION SEMICIRCLE AT (0,0).



SLOPE OF RADIAL RAY: $\Delta y/\Delta x = y/x$

TANGENT IS PERPENDICULAR

SLOPE $= -x/y$.

ANALYTICAL: ALGEBRAIC

AT A POINT $f(x) = x^2$ AT $X = 3$.

$$f'(3) = \lim_{H\to 0} \frac{(3+H)^2-3^2}{H} = \lim_{H\to 0} \frac{9+6H+H^2-9}{H}$$

$$= \lim_{H\to 0} \frac{6H+H^2}{H} = \lim_{H\to 0} \frac{H}{H}(6+H) = \lim_{H\to 0}(6+H) = 6$$

- EVALUATE LIMIT BY REMOVING $\frac{0}{0}$ HOLE.

FUNCTION $f(x) = x^2$ $f'(x) = \lim_{H\to 0} \frac{(x+H)^2-x^2}{H} = \lim_{H\to 0} \frac{2xH+H^2}{H} = \lim_{H\to 0}(2x+H) = 2x$

- FIND FORMULA BY TAKING THE LIMIT USING THE DEFINITION OF $f'(x)$
- USE FORMULAS AS SHORTCUTS. They agree with the LIMIT definition

$f(x)$:	C = CONSTANT	$mx+b$	x^2	x^N	e^x	$\ln(x)$	$\sin(x)$	$\cos(x)$
$f'(x)$:	0	m	$2x$	$N \cdot x^{N-1}$	e^x	$\frac{1}{x}$	$\cos(x)$	$-\sin(x)$

and • USE RULES FOR PRODUCTS, QUOTIENTS, COMPOSITIONS (CHAIN RULE), INVERSES.