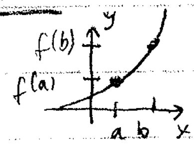
SHEET # 211:
DEFINITIONS AND NOTATION FOR DERIVATIVES AND MOTION

FUNCTIONS IN GENERAL

Let $y = f(x)$

- Average rate of change from $x=a$ to $x=b$:

$$\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$$



- Instantaneous Rate of Change

= Rate of change at $x=a$

= Slope of curve at $x=a$

= Slope of tangent line at $x=a$

= DERIVATIVE at $x=a$:

$$\left.\frac{dy}{dx}\right|_{x=a} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$f'(a) = \lim_{H \to 0} \frac{f(a+H)-f(a)}{H}$$

where $H = \Delta x$.

If limit exists, f is DIFFERENTIABLE at $x=a$.

- DERIVATIVE FUNCTION

$$f'(x) = \frac{df(x)}{dx} = \lim_{H\to 0} \frac{f(x+H)-f(x)}{H}$$

- Difference Quotient

$$\frac{f(x+H)-f(x)}{H}$$

- SECOND DERIVATIVE

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

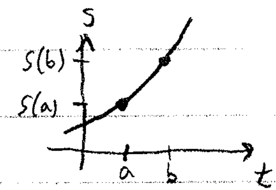
$$f''(x) = \frac{d}{dx} f'(x) = \frac{d^2}{dx^2} f(x)$$

MOTION OF A PARTICLE ALONG A LINE

Let $s(t)$ = POSITION at time t.

- Average velocity from $t=a$ to $t=b$

$$\frac{\Delta s}{\Delta t} = \frac{s(b)-s(a)}{b-a}$$



- Change in time: $\Delta t = b-a$
- DISPLACEMENT from $t=a$ to $t=b$

= Net change in position:

$$\Delta s = s(b)-s(a) = s(a+\Delta t)-s(a)$$

- DISTANCE = total length traveled ≥ 0.
- Instantaneous VELOCITY at $t=a$:

$$v(a) = \left.\frac{ds}{dt}\right|_{t=a} = \lim_{\Delta t\to 0} \frac{\Delta s}{\Delta t}$$

$$v(a) = s'(a) = \lim_{H\to 0} \frac{s(a+H)-s(a)}{H}$$

where $H = \Delta t$.

- SPEED: $|v(a)| \geq 0$
- VELOCITY as a function of time

$$v(t) = s'(t) = \frac{ds}{dt}$$

- ACCELERATION

$$a(t) = s''(t) = \frac{d^2s}{dt^2}$$

$$a(t) = v'(t) = \frac{dv}{dt}$$