SHEET #161:
RATIONAL FUNCTION GRAPHS

NAME: KEY Period: ____

GROUP MEMBERS: _____ (Version 3.)

FOR QUESTIONS 1 AND 2,
SKETCH THESE FUNCTIONS, $f(x)$,
BUT FIRST, FIND (if they exist)

a, ZEROS

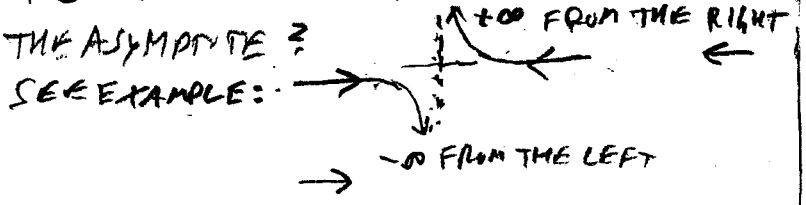
b, y-INTERCEPT

c, VERTICAL ASYMPTOTES

d, HORIZONTAL ASYMPTOTES

e, SLANT ASYMPTOTES

f, THE BEHAVIOR NEAR VERTICAL ASYMPTOTES. DOES $f(x)$ APPROACH $+\infty$ OR $-\infty$ AS x APPROACHES THE ASYMPTOTE?
SEE EXAMPLE:



g, HOLES

1. $f(x) = \dfrac{x-a}{x-2a}$ $(a>0)$

$x-a=0$
$\boxed{x=a}$

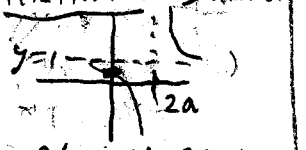
$f(0) = \dfrac{-a}{-2a} = \boxed{\dfrac{1}{2}}$

$x-2a=0$
$\boxed{x=2a}$

$x \to \infty$
$f(x) \to \dfrac{x}{x} \to 1$
$\boxed{y=1}$

NO SLANT

METHOD 1: Sketch



$f(0) = 1/2$ SAYS

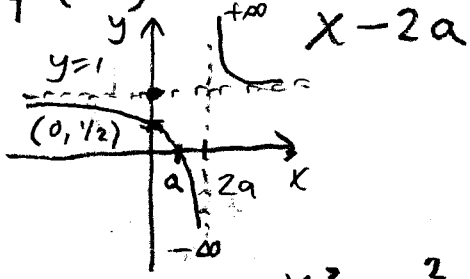
$x \to 2a^-$ (FROM LEFT)
$f(x) \to -\infty$

$x \to 2a^+$ (FROM RIGHT)
$f(x) \to \infty$

METHOD 2:
PLUG IN VALUES SUCH AS
$x = 1.9a$
AND
$x = 2.1a$

$f(1.9a) = [\text{Left}]$
$\dfrac{1.9a-a}{1.9a-2a} =$
$= \dfrac{0.9a}{-0.1a} = -9a$
$\boxed{f \to -\infty}$ as $x \to 2a^-$

$f(2.1a) = [\text{Right}]$
$= \dfrac{2.1a-a}{2.1a-2a} =$
$= \dfrac{1.1a}{0.1a} = 11a$
$\boxed{f \to \infty}$ as $x \to 2a^+$.



2. $f(x) = \dfrac{x^2-a^2}{x-a}$ $(a>0)$

$x^2-a^2=0$
$(x-a)(x+a)=0$
$\boxed{x=-a}$ +SEE BELOW

$f(0) = \dfrac{-a^2}{-a} = \boxed{a}$

$f(x) = \dfrac{(x-a)(x+a)}{x-a}$
$= x+a, \quad x \neq a$

NO ASYMPTOTES

DIVIDE BY 0 FOR $x=a$
↓
(SO NO ZERO AT $x=a$)

HOLE AT
$x=a$
$\boxed{(a, 2a)}$